InspireData™ Standards Match

NEW YORK

Learning Standards for Mathematics, Science and Technology

Meeting curriculum standards is a major focus in education today. This document highlights the correlation of **InspireData™** with the **New York Learning Standards for Mathematics, Science and Technology.**

The Inspired Standards Match is designed to demonstrate the many ways InspireData supports the standards and to give educators ideas for using this tool to meet learning goals.

How to read the InspireData Standards Match:

- **Yellow** highlight indicates a standard or objective that can be supported by the use of InspireData databases, database templates, user generated databases, lesson plans or program features.
- **Green** notes list details about how InspireData can be used to meet the standards, including examples of specific databases, lesson plans or features that support them.

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Learning Standards for

Mathematics, Science, and Technology

Revised Edition
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Learning Standards for Mathematics, Science, and Technology at Three Levels

Standard 1: Students will use mathematical analysis, scientific inquiry, and engineering design, as appropriate, to pose questions, seek answers, and develop solutions.

Standard 2: Students will access, generate, process, and transfer information using appropriate technologies.

Standard 3: Students will understand mathematics and become mathematically confident by communicating and reasoning mathematically, by applying mathematics in real-world settings, and by solving problems through the integrated study of number systems, geometry, algebra, data analysis, probability, and trigonometry.

Standard 4: Students will understand and apply scientific concepts, principles, and theories pertaining to the physical setting and living environment and recognize the historical development of ideas in science.

Standard 5: Students will apply technological knowledge and skills to design, construct, use, and evaluate products and systems to satisfy human and environmental needs.

Standard 6: Students will understand the relationships and common themes that connect mathematics, science, and technology and apply the themes to these and other areas of learning.

Standard 7: Students will apply the knowledge and thinking skills of mathematics, science, and technology to address real-life problems and make informed decisions.

Standard 1—Analysis, Inquiry, and Design

Elementary

Mathematical Analysis

1. Abstraction and symbolic representation are used to communicate mathematically.

Students:

- **use special mathematical notation and symbolism to communicate in mathematics and to compare and describe quantities, express relationships, and relate mathematics to their immediate environments.**

This is evident, for example, when students:

▲ describe their ages as an inequality such as 7 < ❒ < 10.

2. Deductive and inductive reasoning are used to reach mathematical conclusions.

Students:

- **use simple logical reasoning to develop conclusions, recognizing that patterns and relationships present in the environment assist them in reaching these conclusions.**

3. Critical thinking skills are used in the solution of mathematical problems.

Students:

- **explore and solve problems generated from school, home, and community situations, using concrete objects or manipulative materials when possible.**

Scientific Inquiry

1. The central purpose of scientific inquiry is to develop explanations of natural phenomena in a continuing, creative process.

Students:

- **ask "why" questions in attempts to seek greater understanding concerning objects and events they have observed and heard about.**
- **question the explanations they hear from others and read about, seeking clarification and comparing them with their own observations and understandings.**
- **develop relationships among observations to construct descriptions of objects and events and to form their own tentative explanations of what they have observed.**

This is evident, for example, when students:

▲ observe *a variety of objects that either sink or float when placed in a container of water.** Working in groups, they propose an explanation of *why objects sink or float.* After sharing and discussing their proposed explanation, they refine it and submit it for assessment. The explanation is rated on clarity and plausibility.

2. Beyond the use of reasoning and consensus, scientific inquiry involves the testing of proposed explanations involving the use of conventional techniques and procedures and usually requiring considerable ingenuity.

Students:

- **develop written plans for exploring phenomena or for evaluating explanations guided by questions or proposed explanations they have helped formulate.**
- **share their research plans with others and revise them based on their suggestions.**
- **carry out their plans for exploring phenomena through direct observation and through the use of simple instruments that permit measurements of quantities (e.g., length, mass, volume, temperature, and time).**

This is evident, for example, when students:

▲ are asked to develop a way of testing their explanation of *why objects sink or float when placed in a container of water.** They tell what procedures and materials they will use and indicate what results will support their explanation. Their plan is critiqued by others, they revise it, and submit it for assessment. The plan is rated on clarity, soundness in addressing the issue, and feasibility. After the teacher suggests modifications, the plan is carried out.

Key ideas are identified by numbers (1).
Performance indicators are identified by bullets (•).
Sample tasks are identified by triangles (▲).

Students will use mathematical analysis, scientific inquiry, and engineering design, as appropriate, to pose questions, seek answers, and develop solutions.

3. The observations made while testing proposed explanations, when analyzed using conventional and invented methods, provide new insights into phenomena.

Students:

- **organize observations and measurements of objects and events through classification and the preparation of simple charts and tables.**
- **interpret organized observations and measurements, recognizing simple patterns, sequences, and relationships.**
- **share their findings with others and actively seek their interpretations and ideas.**
- **adjust their explanations and understandings of objects and events based on their findings and new ideas.**

This is evident, for example, when students:

▲ prepare tables or other representations of their observations and look for evidence which supports or refutes their explanation of *why objects sink or float when placed in a container of water*.* After sharing and discussing their results with other groups, they prepare a brief research report that includes methods, findings, and conclusions. The report is rated on its clarity, care in carrying out the plan, and presentation of evidence supporting the conclusions.

Engineering Design

1. Engineering design is an iterative process involving modeling and optimization finding the best solution within given constraints which is used to develop technological solutions to problems within given constraints.

Students engage in the following steps in a design process:

- **describe objects, imaginary or real, that might be modeled or made differently and suggest ways in which the objects can be changed, fixed, or improved.**
- **investigate prior solutions and ideas from books, magazines, family, friends, neighbors, and community members.**
- **generate ideas for possible solutions, individually and through group activity; apply age-appropriate mathematics and science skills; evaluate the ideas and determine the best solution; and explain reasons for the choices.**
- **plan and build, under supervision, a model of the solution using familiar materials, processes, and hand tools.**
- **discuss how best to test the solution; perform the test under teacher supervision; record and portray results through numerical and graphic means; discuss orally why things worked or didn't work; and summarize results in writing, suggesting ways to make the solution better.**

This is evident, for example, when students:

▲ read a story called *Humpty's Big Day* wherein the readers visit the place where Humpty Dumpty had his accident, and are asked to design and model a way to get to the top of the wall and down again safely.

▲ generate, draw, and model ideas for a space station that includes a pleasant living and working environment.

▲ design and model footwear that they could use to walk on a cold, sandy surface.

* A variety of content-specific items can be substituted for the italicized text

Standard 1—Analysis, Inquiry, and Design

Intermediate

Mathematical Analysis

1. Abstraction and symbolic representation are used to communicate mathematically.

Students:

- **extend mathematical notation and symbolism to include variables and algebraic expressions in order to describe and compare quantities and express mathematical relationships.**

2. Deductive and inductive reasoning are used to reach mathematical conclusions.

Students:

- **use inductive reasoning to construct, evaluate, and validate conjectures and arguments, recognizing that patterns and relationships can assist in explaining and extending mathematical phenomena.**

This is evident, for example, when students:

▲ predict the next triangular number by examining the pattern 1, 3, 6, 10, ❐.

3. Critical thinking skills are used in the solution of mathematical problems.

Students:

- **apply mathematical knowledge to solve real-world problems and problems that arise from the investigation of mathematical ideas, using representations such as pictures, charts, and tables.**

Key ideas are identified by numbers (1).
Performance indicators are identified by bullets (•).
Sample tasks are identified by triangles (▲).

Scientific Inquiry

1. The central purpose of scientific inquiry is to develop explanations of natural phenomena in a continuing, creative process.

Students:

- **formulate questions independently with the aid of references appropriate for guiding the search for explanations of everyday observations.**
- **construct explanations independently for natural phenomena, especially by proposing preliminary visual models of phenomena.**
- **represent, present, and defend their proposed explanations of everyday observations so that they can be understood and assessed by others.**
- **seek to clarify, to assess critically, and to reconcile with their own thinking the ideas presented by others, including peers, teachers, authors, and scientists.**

This is evident, for example, when students:

▲ *After being shown the disparity between the amount of solid waste which is recycled and which could be recycled,** students working in small groups are asked to explain why this disparity exists. They develop a set of possible explanations and to select one for intensive study. After their explanation is critiqued by other groups, it is refined and submitted for assessment. The explanation is rated on clarity, plausibility, and appropriateness for intensive study using research methods.

2. Beyond the use of reasoning and consensus, scientific inquiry involves the testing of proposed explanations involving the use of conventional techniques and procedures and usually requiring considerable ingenuity.

Students:

- **use conventional techniques and those of their own design to make further observations and refine their explanations, guided by a need for more information.**
- **develop, present, and defend formal research proposals for testing their own explanations of common phenomena, including ways of obtaining needed observations and ways of conducting simple controlled experiments.**
- **carry out their research proposals, recording observations and measurements (e.g., lab notes, audio tape, computer disk, video tape) to help assess the explanation.**

This is evident, for example, when students:

▲ develop a research plan for studying the accuracy of their explanation of the *disparity between the amount of solid waste that is recycled and that could be recycled.** After their tentative plan is critiqued, they refine it and submit it for assessment. The research proposal is rated on clarity, feasibility and soundness as a method of studying the explanations' accuracy. They carry out the plan, with teacher suggested modifications. This work is rated by the teacher while it is in progress.

Students will use mathematical analysis, scientific inquiry, and engineering design, as appropriate, to pose questions, seek answers, and develop solutions.

3. The observations made while testing proposed explanations, when analyzed using conventional and invented methods, provide new insights into phenomena.

Students:

- **design charts, tables, graphs and other representations of observations in conventional and creative ways to help them address their research question or hypothesis.**
- **interpret the organized data to answer the research question or hypothesis and to gain insight into the problem.**
- **modify their personal understanding of phenomena based on evaluation of their hypothesis.**

This is evident, for example, when students:

▲ carry out their plan making appropriate observations and measurements. They analyze the data, reach conclusions regarding their explanation of the *disparity between the amount of solid waste which is recycled and which could be recycled.**, and prepare a tentative report which is critiqued by other groups, refined, and submitted for assessment. The report is rated on clarity, quality of presentation of data and analyses, and soundness of conclusions.

Engineering Design

1. Engineering design is an iterative process involving modeling and optimization finding the best solution within given constraints which is used to develop technological solutions to problems within given constraints.

Students engage in the following steps in a design process:

- **identify needs and opportunities for technical solutions from an investigation of situations of general or social interest.**
- **locate and utilize a range of printed, electronic, and human information resources to obtain ideas.**
- **consider constraints and generate several ideas for alternative solutions, using group and individual ideation techniques (group discussion, brainstorming, forced connections, role play); defer judgment until a number of ideas have been generated; evaluate (critique) ideas; and explain why the chosen solution is optimal.**
- **develop plans, including drawings with measurements and details of construction, and construct a model of the solution, exhibiting a degree of craftsmanship.**
- **in a group setting, test their solution against design specifications, present and evaluate results, describe how the solution might have been modified for different or better results, and discuss tradeoffs that might have to be made.**

This is evident, for example, when students:

▲ reflect on the need for alternative growing systems in desert environments and design and model a hydroponic greenhouse for growing vegetables without soil.

▲ brainstorm and evaluate alternative ideas for an adaptive device that will make life easier for a person with a disability, such as a device to pick up objects from the floor.

▲ design a model vehicle (with a safety belt restraint system and crush zones to absorb impact) to carry a raw egg as a passenger down a ramp and into a barrier without damage to the egg.

▲ assess the performance of a solution against various design criteria, enter the scores on a spreadsheet, and see how varying the solution might have affected total score.

* A variety of content-specific items can be substituted for the italicized text

Standard 1—Analysis, Inquiry, and Design

Commencement

Mathematical Analysis

1. Abstraction and symbolic representation are used to communicate mathematically.

Students:

- **use algebraic and geometric representations to describe and compare data.**

2. Deductive and inductive reasoning are used to reach mathematical conclusions.

Students:

- **use deductive reasoning to construct and evaluate conjectures and arguments, recognizing that patterns and relationships in mathematics assist them in arriving at these conjectures and arguments.**

3. Critical thinking skills are used in the solution of mathematical problems.

Students:

- **apply algebraic and geometric concepts and skills to the solution of problems.**

Key ideas are identified by numbers (1).
Performance indicators are identified by bullets (•).
Sample tasks are identified by triangles (▲).

Scientific Inquiry

1. The central purpose of scientific inquiry is to develop explanations of natural phenomena in a continuing, creative process.

Students:

- **elaborate on basic scientific and personal explanations of natural phenomena, and develop extended visual models and mathematical formulations to represent their thinking.**
- **hone ideas through reasoning, library research, and discussion with others, including experts.**
- **work toward reconciling competing explanations; clarifying points of agreement and disagreement.**
- **coordinate explanations at different levels of scale, points of focus, and degrees of complexity and specificity and recognize the need for such alternative representations of the natural world.**

This is evident, for example, when students:

▲ in small groups, are asked to explain *why a cactus plant requires much less water to survive than many other plants.** They are asked to develop, through research, a set of explanations for the differences and to select at least one for study. After the proposed explanation is critiqued by others, they refine it by formulating a hypothesis which is rated on clarity, plausibility, and researchability.

2. Beyond the use of reasoning and consensus, scientific inquiry involves the testing of proposed explanations involving the use of conventional techniques and procedures and usually requiring considerable ingenuity.

Students:

- **devise ways of making observations to test proposed explanations.**
- **refine their research ideas through library investigations, including electronic information retrieval and reviews of the literature, and through peer feedback obtained from review and discussion.**
- **develop and present proposals including formal hypotheses to test their explanations, i.e., they predict what should be observed under specified conditions if the explanation is true.**
- **carry out their research plan for testing explanations, including selecting and developing techniques, acquiring and building apparatus, and recording observations as necessary.**

This is evident, for example, when students:

▲ develop, through research, a proposal to test their hypothesis *of why a cactus plant requires much less water to survive than many other plants.** After their proposal is critiqued, it is refined and submitted for assessment by a panel of students. The proposal is rated on clarity, appropriateness, and feasibility. Upon approval, students complete the research. Progress is rated holistically by the teacher.

Students will use mathematical analysis, scientific inquiry, and engineering design, as appropriate, to pose questions, seek answers, and develop solutions.

3. The observations made while testing proposed explanations, when analyzed using conventional and invented methods, provide new insights into phenomena.

Students:

- **use various means of representing and organizing observations (e.g., diagrams, tables, charts, graphs, equations, matrices) and insightfully interpret the organized data.**
- **apply statistical analysis techniques when appropriate to test if chance alone explains the result.**
- **assess correspondence between the predicted result contained in the hypothesis and the actual result and reach a conclusion as to whether or not the explanation on which the prediction was based is supported.**
- **based on the results of the test and through public discussion, they revise the explanation and contemplate additional research.**
- **develop a written report for public scrutiny that describes their proposed explanation, including a literature review, the research they carried out, its result, and suggestions for further research.**

This is evident, for example, when students:

▲ carry out a research plan, including keeping a lab book, to test their hypothesis *of why a cactus plant requires much less water to survive than many other plants.** After completion, a paper is presented describing the research. Based on the class critique, the paper is rewritten and submitted with the lab book for separate assessment or as part of a portfolio of their science work. It is rated for clarity, thoroughness, soundness of conclusions, and quality of integration with existing literature.

Engineering Design

1. Engineering design is an iterative process involving modeling and optimization finding the best solution within given constraints which is used to develop technological solutions to problems within given constraints.

Students engage in the following steps in a design process:

- **initiate and carry out a thorough investigation of an unfamiliar situation and identify needs and opportunities for technological invention or innovation.**
- **identify, locate, and use a wide range of information resources, and document through notes and sketches how findings relate to the problem.**
- **generate creative solutions, break ideas into significant functional elements, and explore possible refinements; predict possible outcomes using mathematical and functional modeling techniques; choose the optimal solution to the problem, clearly documenting ideas against design criteria and constraints; and explain how human understands, economics, ergonomics, and environmental considerations have influenced the solution.**
- **develop work schedules and working plans which include optimal use and cost of materials, processes, time, and expertise; construct a model of the solution, incorporating developmental modifications while working to a high degree of quality (craftsmanship).**
- **devise a test of the solution according to the design criteria and perform the test; record, portray, and logically evaluate performance test results through quantitative, graphic, and verbal means. Use a variety of creative verbal and graphic techniques effectively and persuasively to present conclusions, predict impacts and new problems, and suggest and pursue modifications.**

This is evident, for example, when students:

▲ search the Internet for world wide web sites dealing with renewable energy and sustainable living and research the development and design of an energy efficient home.

▲ develop plans, diagrams, and working drawings for the construction of a computer-controlled marble sorting system that simulates how parts on an assembly line are sorted by color.

▲ design and model a portable emergency shelter that could be heated by a person's body to a life-sustaining temperature when the outside temperature is 20^{o} F.

* A variety of content-specific items can be substituted for the italicized text

Standard 2—Information Systems
Elementary

Information Systems

1. Information technology is used to retrieve, process, and communicate information and as a tool to enhance learning.

Students:
- **use a variety of equipment and software packages to enter, process, display, and communicate information in different forms using text, tables, pictures, and sound.**
- **telecommunicate a message to a distant location with teacher help.**
- **access needed information from printed media, electronic data bases, and community resources.**

This is evident, for example, when students:

▲ use the newspaper or magazine index in a library to find information on a particular topic.
▲ invite local experts to the school to share their expertise.

2. Knowledge of the impacts and limitations of information systems is essential to its effective and ethical use.

Students:
- **describe the uses of information systems in homes, schools, and businesses.**
- **understand that computers are used to store personal information.**
- **demonstrate ability to evaluate information.**

This is evident, for example, when students:

▲ look for differences among species of bugs collected on the school grounds, and classify them according to preferred habitat.

Key ideas are identified by numbers (1).
Performance indicators are identified by bullets (•).
Sample tasks are identified by triangles (▲).

Students will access, generate, process, and transfer information using appropriate technologies.

3. Information technology can have positive and negative impacts on society, depending upon how it is used.

Students:

- **describe the uses of information systems in homes and schools.**
- **demonstrate ability to evaluate information critically.**

Standard 2—Information Systems

Intermediate

Information Systems

1. Information technology is used to retrieve, process, and communicate information and as a tool to enhance learning.

Students:

- **use a range of equipment and software to integrate several forms of information in order to create good quality audio, video, graphic, and text-based presentations.**
- **use spreadsheets and data-base software to collect, process, display, and analyze information. Students access needed information from electronic data bases and on-line telecommunication services.**
- **systematically obtain accurate and relevant information pertaining to a particular topic from a range of sources, including local and national media, libraries, museums, governmental agencies, industries, and individuals.**
- **collect data from probes to measure events and phenomena.**
- **use simple modeling programs to make predictions.**

This is evident, for example, when students:

▲ compose letters on a word processor and send them to representatives of industry, governmental agencies, museums, or laboratories seeking information pertaining to a student project.
▲ acquire data from weather stations.
▲ use a software package, such as Science Tool Kit, to monitor the acceleration of a model car traveling down a given distance on a ramp.
▲ use computer software to model how plants grow plants under different conditions.

2. Knowledge of the impacts and limitations of information systems is essential to its effective and ethical use.

Students:

- **understand the need to question the accuracy of information displayed on a computer because the results produced by a computer may be affected by incorrect data entry.**
- **identify advantages and limitations of data-handling programs and graphics programs.**
- **understand why electronically stored personal information has greater potential for misuse than records kept in conventional form.**

Key ideas are identified by numbers (1).
Performance indicators are identified by bullets (•).
Sample tasks are identified by triangles (▲).

Students will access, generate, process, and transfer information using appropriate technologies.

3. Information technology can have positive and negative impacts on society, depending upon how it is used.

Students:

- **use graphical, statistical, and presentation software to presents project to fellow classmates.**
- **describe applications of information technology in mathematics, science, and other technologies that address needs and solve problems in the community.**
- **explain the impact of the use and abuse of electronically generated information on individuals and families.**

Standard 2—Information Systems

Commencement

Information Systems

1. Information technology is used to retrieve, process, and communicate information and as a tool to enhance learning.

Students:

- **understand and use the more advanced features of word processing, spreadsheets, and data-base software.**
- **prepare multimedia presentations demonstrating a clear sense of audience and purpose.**
- **access, select, collate, and analyze information obtained from a wide range of sources such as research data bases, foundations, organizations, national libraries, and electronic communication networks, including the Internet.**
- **students receive news reports from abroad and work in groups to produce newspapers reflecting the perspectives of different countries.**
- **utilize electronic networks to share information.**
- **model solutions to a range of problems in mathematics, science, and technology using computer simulation software.**

This is evident, for example, when students:

▲ collect and amend quantitative and qualitative information for a particular purpose and enter it into a data-handling package for processing and analysis.

▲ visit businesses, laboratories, environmental areas, and universities to obtain on-site information

▲ recieve news reports from abroad, and work in groups to produce newspapers reflecting the perspectives of different countries.

▲ join a list serve and send electronic mail to other persons sharing mutual concerns and interests.

▲ use computer software to simulate and graph the motion of an object.

▲ study a system in a dangerous setting (e.g., a nuclear power plant).

2. Knowledge of the impacts and limitations of information systems is essential to its effective and ethical use.

Students:

- **explain the impact of the use and abuse of electronically generated information on individuals and families.**
- **evaluate software packages relative to their suitability to a particular application and their ease of use.**
- **discuss the ethical and social issues raised by the use and abuse of information systems.**

This is evident, for example, when students:

▲ discuss how unauthorized people might gain access to information about their interests and way of life.

Key ideas are identified by numbers (1).
Performance indicators are identified by bullets (•).
Sample tasks are identified by triangles (▲).

Students will access, generate, process, and transfer information using appropriate technologies.

3. Information technology can have positive and negative impacts on society, depending upon how it is used.

Students:

- **work with a virtual community to conduct a project or solve a problem using the network.**
- **discuss how applications of information technology can address some major global problems and issues.**
- **discuss the environmental, ethical, moral, and social issues raised by the use and abuse of information technology.**

Standard 3—Mathematics

Elementary

Mathematical Reasoning

1. Students use mathematical reasoning to analyze mathematical situations, make conjectures, gather evidence, and construct an argument.

Students:

- **use models, facts, and relationships to draw conclusions about mathematics and explain their thinking.**
- **use patterns and relationships to analyze mathematical situations.**
- **justify their answers and solution processes.**
- **use logical reasoning to reach simple conclusions.**

This is evident, for example, when students:

▲ build geometric figures out of straws.

▲ find patterns in sequences of numbers, such as the triangular numbers 1, 3, 6, 10, ...

▲ explore number relationships with a calculator (e.g., 12 + 6 = 18, 11 + 7 = 18, etc.) and draw conclusions.

Number and Numeration

2. Students use number sense and numeration to develop an understanding of the multiple uses of numbers in the real world, the use of numbers to communicate mathematically, and the use of numbers in the development of mathematical ideas.

Students:

- **use whole numbers and fractions to identify locations, quantify groups of objects, and measure distances.**
- **use concrete materials to model numbers and number relationships for whole numbers and common fractions, including decimal fractions.**
- **relate counting to grouping and to place-value.**
- **recognize the order of whole numbers and commonly used fractions and decimals.**
- **demonstrate the concept of percent through problems related to actual situations.**

This is evident, for example, when students:

▲ count out 15 small cubes and exchange ten of the cubes for a rod ten cubes long.

▲ use the number line to show the position of 1/4.

▲ figure the tax on $4.00 knowing that taxes are 7 cents per $1.00.

Sample Problems

16. Marlene is designing a uniform for her soccer team. She can choose from 2 different shirts and 3 different pairs of shorts. How many different uniforms can she make if she uses all the shirts and all the shorts?

Answer ______________________

Explain how you got your answer with a picture or diagram.

Ms. Rivera's class must collect 180 soda cans to win the recycling contest. The chart below shows how the class is doing. How many cans must they collect in the fourth week to reach the goal of 180?

Week	Cans
1	42
2	74
3	18
4	
Goal	180

Answer ____________________

Key ideas are identified by numbers (1).
Performance indicators are identified by bullets (•).
Sample tasks are identified by triangles (▲).

Students will understand mathematics and become mathematically confident by communicating and reasoning mathematically, by applying mathematics in real-world settings, and by solving problems through the integrated study of number systems, geometry, algebra, data analysis, probability, and trigonometry.

Operations

3. Students use mathematical operations and relationships among them to understand mathematics.

Students:

- **add, subtract, multiply, and divide whole numbers.**
- **develop strategies for selecting the appropriate computational and operational method in problem-solving situations.**
- **know single digit addition, subtraction, multiplication, and division facts.**
- **understand the commutative and associative properties.**

This is evident, for example, when students:

▲ use the fact that multiplication is commutative (e.g., 2 x 7 = 7 x 2), to assist them with their memorizing of the basic facts.

▲ solve multiple-step problems that require at least two different operations.

▲ progress from base ten blocks to concrete models and then to paper and pencil algorithms.

Modeling/Multiple Representation

4. Students use mathematical modeling/multiple representation to provide a means of presenting, interpreting, communicating, and connecting mathematical information and relationships.

Students:

- **use concrete materials to model spatial relationships.**
- **construct tables, charts, and graphs to display and analyze real-world data.**
- **use multiple representations (simulations, manipulative materials, pictures, and diagrams) as tools to explain the operation of everyday procedures.**
- **use variables such as height, weight, and hand size to predict changes over time.**
- **use physical materials, pictures, and diagrams to explain mathematical ideas and processes and to demonstrate geometric concepts.**

This is evident, for example, when students:

▲ build a 3 x 3 x 3 cube out of blocks.

▲ use square tiles to model various rectangles with an area of 24 square units.

▲ read a bar graph of population trends and write an explanation of the information it contains.

Sample Problems

7. Shanelle earns $3.50 per hour for babysitting. Each week she babysits for 4 hours.

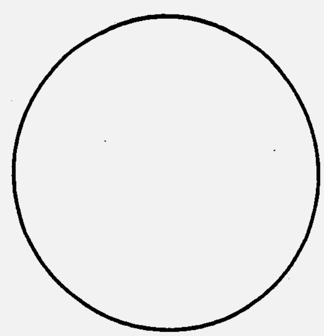
A) How much money does she earn in 1 week?

Answer ________________

B) How much money does she earn in 4 weeks?

Answer ________________

11. Bobbie's family bought a pizza. Her mother and sister together ate $\frac{1}{2}$ of the pizza. Bobbie ate $\frac{1}{2}$ of what was left. Use the circle to draw a picture that shows how much of the pizza Bobbi ate.



What fraction of the whole pizza did Bobbie eat?

Answer ________

Standard 3—Mathematics
Elementary

Measurement

5. Students use measurement in both metric and English measure to provide a major link between the abstractions of mathematics and the real world in order to describe and compare objects and data.

Students:

- **understand that measurement is approximate, never exact.**
- **select appropriate standard and nonstandard measurement tools in measurement activities.**
- **understand the attributes of area, length, capacity, weight, volume, time, temperature, and angle.**
- **estimate and find measures such as length, perimeter, area, and volume using both nonstandard and standard units.**
- **collect and display data.**
- **use statistical methods such as graphs, tables, and charts to interpret data.**

This is evident, for example, when students:

▲ measure with paper clips or finger width.

▲ estimate, then calculate, how much paint would be needed to cover one wall.

▲ create a chart to display the results of a survey conducted among the classes in the school, or graph the amounts of survey responses by grade level.

Uncertainty

6. Students use ideas of uncertainty to illustrate that mathematics involves more than exactness when dealing with everyday situations.

Students:

- **make estimates to compare to actual results of both formal and informal measurement.**
- **make estimates to compare to actual results of computations.**
- **recognize situations where only an estimate is required.**
- **develop a wide variety of estimation skills and strategies.**
- **determine the reasonableness of results.**
- **predict experimental probabilities.**
- **make predictions using unbiased random samples.**
- **determine probabilities of simple events.**

This is evident, for example, when students:

▲ estimate the length of the room before measuring.

▲ predict the average number of red candies in a bag before opening a group of bags, counting the candies, and then averaging the number that were red.

▲ determine the probability of picking an even numbered slip from a hat containing slips of paper numbered 1, 2, 3, 4, 5, and 6.

Sample Problems

It's Saturday and you're going to meet your friends for lunch and a movie. You have to leave your home at 11:30 AM. Your parents say you can't go until you finish your work. Your work includes your homework and your Saturday chores:

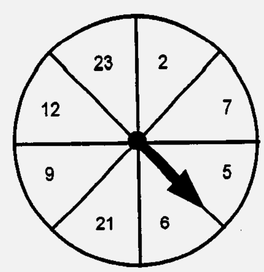
40 minutes of math homework.
30 minutes to clean your room.
15 minutes to fold the laundry
5 minutes to take out the garbage
60 minutes to eat and get ready to go

A) At what time should you get started doing your work? Show all the math you did to figure this out.

Answer ____________________ AM

B) Describe how you would use your time between when you wake up and when you leave at 11:30 AM to go to lunch and the movie.

The spinner below was used by Jodie's class for the school fair:



A) If the spinner is spun once, what is the probability of the spinner landing on an even number?

Answer $\frac{\quad}{\quad}$

B) If the spinner is spun a second time, what is the probability of the spinner landing on a number that is divisible by 3?

Answer $\frac{\quad}{\quad}$

Key ideas are identified by numbers (1).
Performance indicators are identified by bullets (•).
Sample tasks are identified by triangles (▲).

Students will understand mathematics and become mathematically confident by communicating and reasoning mathematically, by applying mathematics in real-world settings, and by solving problems through the integrated study of number systems, geometry, algebra, data analysis, probability, and trigonometry.

Patterns/Functions

7. Students use patterns and functions to develop mathematical power, appreciate the true beauty of mathematics, and construct generalizations that describe patterns simply and efficiently.

Students:

- **recognize, describe, extend, and create a wide variety of patterns.**
- **represent and describe mathematical relationships.**
- **explore and express relationships using variables and open sentences.**
- **solve for an unknown using manipulative materials.**
- **use a variety of manipulative materials and technologies to explore patterns.**
- **interpret graphs.**
- **explore and develop relationships among two- and three-dimensional geometric shapes.**
- **discover patterns in nature, art, music, and literature.**

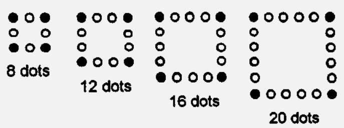
This is evident, for example, when students:

▲ represent three more than a number is equal to nine as n + 3 = 9.

▲ draw leaves, simple wallpaper patterns, or write number sequences to illustrate recurring patterns.

▲ write generalizations or conclusions from display data in charts or graphs.

Sample Problem



Draw the next figure in this pattern. How many dots are in the figure you drew?

Answer ____________________

Write one or two sentences to describe how the figure is changing.

____

____

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Standard 3—Mathematics

Intermediate

Mathematical Reasoning

1. Students use mathematical reasoning to analyze mathematical situations, make conjectures, gather evidence, and construct an argument.

Students:

- **apply a variety of reasoning strategies.**
- **make and evaluate conjectures and arguments using appropriate language.**
- **make conclusions based on inductive reasoning.**
- **justify conclusions involving simple and compound (i.e., and/or) statements.**

This is evident, for example, when students:

▲ use trial and error and work backwards to solve a problem.
▲ identify patterns in a number sequence.
▲ are asked to find numbers that satisfy two conditions, such as $n > -4$ and $n \leq 6$.

Number and Numeration

2. Students use number sense and numeration to develop an understanding of the multiple uses of numbers in the real world, the use of numbers to communicate mathematically, and the use of numbers in the development of mathematical ideas.

Students:

- **understand, represent, and use numbers in a variety of equivalent forms (integer, fraction, decimal, percent, exponential, expanded and scientific notation).**
- **understand and apply ratios, proportions, and percents through a wide variety of hands-on explorations.**
- **develop an understanding of number theory (primes, factors, and multiples).**
- **recognize order relations for decimals, integers, and rational numbers.**

This is evident, for example, when students:

▲ use prime factors of a group of denominators to determine the least common denominator.
▲ select two pairs from a number of ratios and prove that they are in proportion.
▲ demonstrate the concept that a number can be symbolized by many different numerals as in:

$$\frac{1}{4} = \frac{3}{12} = \frac{25}{100} = 0.25 = 25\%$$

Sample Problems

The table below shows the height of a plant during a period of 3 weeks. Initially the plant was 5 inches tall. The table indicates the growth rate of the plant for week 1 through week 3.

Weeks (W)	0	1	2	3
Height (H) (in inches)	5	8	11	14

A) Write an equation that expresses the height (H) of the plant in terms of the number of weeks (W).

Answer: ___________________

B) Use the table or your equation to predict the height of the plant after 10 weeks.

2. An inspector found 5 defective cassettes out of a random sample of 200 cassette tapes. If 4,000 cassette tapes are produced each day, how many tapes would you expect to be defective? Write a proportion that can be used to solve this problem and then solve the problem.

Answer: ___________________

Key ideas are identified by numbers (1).
Performance indicators are identified by bullets (•).
Sample tasks are identified by triangles (▲).

Students will understand mathematics and become mathematically confident by communicating and reasoning mathematically, by applying mathematics in real-world settings, and by solving problems through the integrated study of number systems, geometry, algebra, data analysis, probability, and trigonometry.

Operations

3. Students use mathematical operations and relationships among them to understand mathematics.

Students:

- **add, subtract, multiply, and divide fractions, decimals, and integers.**
- **explore and use the operations dealing with roots and powers.**
- **use grouping symbols (parentheses) to clarify the intended order of operations.**
- **apply the associative, commutative, distributive, inverse, and identity properties.**
- **demonstrate an understanding of operational algorithms (procedures for adding, subtracting, etc.).**
- **develop appropriate proficiency with facts and algorithms.**
- **apply concepts of ratio and proportion to solve problems.**

This is evident, for example, when students:

▲ create area models to help in understanding fractions, decimals, and percents.

▲ find the missing number in a proportion in which three of the numbers are known, and letters are used as place holders.

▲ arrange a set of fractions in order, from the smallest to the largest:

$$\frac{3}{4}, \frac{1}{5}, \frac{2}{3}, \frac{1}{2}, \frac{1}{4}$$

▲ illustrate the distributive property for multiplication over addition, such as

$2(a + 3) = 2a + 6.$

Modeling/Multiple Representation

4. Students use mathematical modeling/multiple representation to provide a means of presenting, interpreting, communicating, and connecting mathematical information and relationships.

Students:

- **visualize, represent, and transform two- and three-dimensional shapes.**
- **use maps and scale drawings to represent real objects or places.**
- **use the coordinate plane to explore geometric ideas.**
- **represent numerical relationships in one- and two-dimensional graphs.**
- **use variables to represent relationships.**
- **use concrete materials and diagrams to describe the operation of real world processes and systems.**
- **develop and explore models that do and do not rely on chance.**
- **investigate both two- and three-dimensional transformations.**
- **use appropriate tools to construct and verify geometric relationships.**
- **develop procedures for basic geometric constructions.**

This is evident, for example, when students:

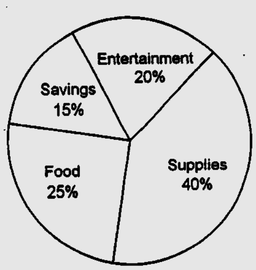
▲ build a city skyline to demonstrate skill in linear measurements, scale drawing, ratio, fractions, angles, and geometric shapes.

▲ bisect an angle using a straight edge and compass.

▲ draw a complex of geometric figures to illustrate that the intersection of a plane and a sphere is a circle or point.

Sample Problems

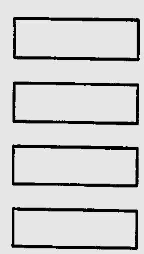
. The graph below shows how Sue spent her allowance last week.



If Sue's allowance is $6.00, how much of her allowance did she spend on entertainment last week?

TASK: SHARING

5. Six students were given four candy bars of equal size. Show how they could divide the candy bars so that each of them received the same amount of candy. Then use the numbers to express how much of a candy bar each student received.



Standard 3—Mathematics

Intermediate

Measurement

5. Students use measurement in both metric and English measure to provide a major link between the abstractions of mathematics and the real world in order to describe and compare objects and data.

Students:

- **estimate, make, and use measurements in real-world situations.**
- **select appropriate standard and nonstandard measurement units and tools to measure to a desired degree of accuracy.**
- **develop measurement skills and informally derive and apply formulas in direct measurement activities.**
- **use statistical methods and measures of central tendencies to display, describe, and compare data.**
- **explore and produce graphic representations of data using calculators/computers.**
- **develop critical judgment for the reasonableness of measurement.**

This is evident, for example, when students:

- ▲ use box plots or stem and leaf graphs to display a set of test scores.
- ▲ estimate and measure the surface areas of a set of gift boxes in order to determine how much wrapping paper will be required.
- ▲ explain when to use mean, median, or mode for a group of data.

Uncertainty

6. Students use ideas of uncertainty to illustrate that mathematics involves more than exactness when dealing with everyday situations.

Students:

- **use estimation to check the reasonableness of results obtained by computation, algorithms, or the use of technology.**
- **use estimation to solve problems for which exact answers are inappropriate.**
- **estimate the probability of events.**
- **use simulation techniques to estimate probabilities.**
- **determine probabilities of independent and mutually exclusive events.**

This is evident, for example, when students:

- ▲ construct spinners to represent random choice of four possible selections.
- ▲ perform probability experiments with independent events (e.g., the probability that the head of a coin will turn up, or that a 6 will appear on a die toss).
- ▲ estimate the number of students who might chose to eat hot dogs at a picnic.

Sample Problems

TASK: Donello's Pizzeria

1. Donello's is considering adding a 12" in diameter "large" pizza to its menu. One customer says that adding the large size pizza is unnecessary because it is the same amount of pizza as 2 of the 6" size pizzas. Use mathematics to determine if the customer is correct. Show your work and write a few sentences to explain your answer.

Answer ____________________

____
____
____
____
____

TASK: PAY PLANS

You have just gotten an after school job at City Outfitters. This company offers two different payment plans to its sales employees.

Plan A Earnings:	$110 per week plus 10% of sales
Plan B Earnings:	$80 per week plus 15% of sales

You need to decide which plan to choose and explain why you made this choice.

28. To help you decide, you ask the sales manager what the average weekly sales are. She tells you sales vary a lot, but average around $350 a week. How much would you expect to earn under each payment plan during an average week?

Answer: Plan A ____________________

Plan B ____

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Patterns/Functions

7. Students use patterns and functions to develop mathematical power, appreciate the true beauty of mathematics, and construct generalizations that describe patterns simply and efficiently.

Students:

- **recognize, describe, and generalize a wide variety of patterns and functions.**
- **describe and represent patterns and functional relationships using tables, charts and graphs, algebraic expressions, rules, and verbal descriptions.**
- **develop methods to solve basic linear and quadratic equations.**
- **develop an understanding of functions and functional relationships: that a change in one quantity (variable) results in change in another.**
- **verify results of substituting variables.**
- **apply the concept of similarity in relevant situations.**
- **use properties of polygons to classify them.**
- **explore relationships involving points, lines, angles, and planes.**
- **develop and apply the Pythagorean principle in the solution of problems.**
- **explore and develop basic concepts of right triangle trigonometry.**
- **use patterns and functions to represent and solve problems.**

This is evident, for example, when students:

▲ find the height of a building when a 20-foot ladder reaches the top of the building when its base is 12 feet away from the structure.

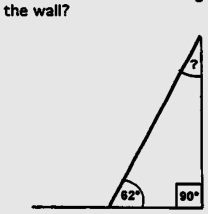
▲ investigate number patterns through palindromes (pick a 2-digit number, reverse it and add the two—repeat the process until a palindrome appears)

	42		86
	+24		+68
palindrome →	66		154
			+451
			605
			+506
		palindrome →	1111

▲ solve linear equations, such as 2(x + 3) = x + 5 by several methods.

Sample Problem

. A painter leaned a ladder up against the wall of my kitchen. The ladder forms an angle of 62° with the floor. What is the measure of the angle formed between the top of the ladder and the wall?



Answer: ____________________

Standard 3—Mathematics

Commencement

Mathematical Reasoning

1. Students use mathematical reasoning to analyze mathematical situations, make conjectures, gather evidence, and construct an argument.

Students:

- **construct simple logical arguments.**
- **follow and judge the validity of logical arguments.**
- **use symbolic logic in the construction of valid arguments.**
- **construct proofs based on deductive reasoning.**

This is evident, for example, when students:

▲ prove that an altitude of an isosceles triangle, drawn to the base, is perpendicular to that base.
▲ determine whether or not a given logical sentence is a tautology.
▲ show that the triangle having vertex coordinates of (0,6), (0,0), and (5,0) is a right triangle.

Sample Problems

33 Given the true statements:

$\sim a \vee \sim b$
b
$c \rightarrow a$

Which statement is also true?

(1) c (3) $\sim c$
(2) $\sim b$ (4) a

34 Which statement is logically equivalent to the statement: "If you are not part of the solution, then you are part of the problem"?

(1) If you are part of the solution, then you are not part of the problem.
(2) If you are not part of the problem, then you are part of the solution.
(3) If you are part of the problem, then you are not part of the solution.
(4) If you are not part of the problem, then you are not part of the solution.

Number and Numeration

2. Students use number sense and numeration to develop an understanding of the multiple uses of numbers in the real world, the use of numbers to communicate mathematically, and the use of numbers in the development of mathematical ideas.

Students:

- **understand and use rational and irrational numbers.**
- **recognize the order of the real numbers.**
- **apply the properties of the real numbers to various subsets of numbers.**

This is evident, for example, when students:

▲ determine from the discriminate of a quadratic equation whether the roots are rational or irrational.
▲ give rational approximations of irrational numbers to a specific degree of accuracy.
▲ determine for which value of x the expression $\frac{2x + 6}{x - 7}$ is undefined.

Key ideas are identified by numbers (1).
Performance indicators are identified by bullets (•).
Sample tasks are identified by triangles (▲).

Students will understand mathematics and become mathematically confident by communicating and reasoning mathematically, by applying mathematics in real-world settings, and by solving problems through the integrated study of number systems, geometry, algebra, data analysis, probability, and trigonometry.

Operations

3. Students use mathematical operations and relationships among them to understand mathematics.

Students:

- **use addition, subtraction, multiplication, division, and exponentiation with real numbers and algebraic expressions.**
- **develop an understanding of and use the composition of functions and transformations.**
- **explore and use negative exponents on integers and algebraic expressions.**
- **use field properties to justify mathematical procedures.**
- **use transformations on figures and functions in the coordinate plane.**

This is evident, for example, when students:

▲ determine the coordinates of triangle A(2,5), B(9,8), and C(3,6) after a translation $(x,y) \rightarrow (x + 3, y - 1)$.

▲ evaluate the binary operation defined as $x * y = x^{-2} + (y + x)^2$ for $3 * 4$.

▲ identify the field properties used in solving the equation $2(x - 5) + 3 = x + 7$.

Modeling/Multiple Representation

4. Students use mathematical modeling/multiple representation to provide a means of presenting, interpreting, communicating, and connecting mathematical information and relationships.

Students:

- **represent problem situations symbolically by using algebraic expressions, sequences, tree diagrams, geometric figures, and graphs.**
- **manipulate symbolic representations to explore concepts at an abstract level.**
- **choose appropriate representations to facilitate the solving of a problem.**
- **use learning technologies to make and verify geometric conjectures .**
- **justify the procedures for basic geometric constructions.**
- **investigate transformations in the coordinate plane.**
- **develop meaning for basic conic sections.**
- **develop and apply the concept of basic loci to compound loci.**
- **use graphing utilities to create and explore geometric and algebraic models.**
- **model real-world problems with systems of equations and inequalities.**

This is evident, for example, when students:

▲ determine the locus of points equidistant from two parallel lines.

▲ explain why the basic construction of bisecting a line is valid.

▲ describe the various conics produced when the equation $ax^2 + by^2 = c^2$ is graphed for various values of a, b, and c.

Sample Problems

36 *a* On graph paper, draw the graph of the equation $y = x^2 - 4x + 3$, including all values of x in the interval $-1 \leq x \leq 5$. [4]

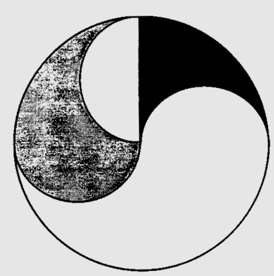
b On the same set of axes, draw the graph of the image of the graph drawn in part *a* after the translation which moves (x,y) to $(x + 3, y + 2)$, and label this graph *b*. [3]

c On the same set of axes, draw the graph of the image of the graph drawn in part *b* after a reflection in the x-axis, and label this graph *c*. [3]

Semicircles

The figure below is made of three small semicircles, all of the same size, and one large circle.

The *diameters* of the semicircles are the same length as the *radius* of the large circle.



Assume that the radius of the large circle is 4 cm long. What is the area of the gray region?

Describe your method: how did you figure it out?

Standard 3—Mathematics

Commencement

Measurement

5. Students use measurement in both metric and English measure to provide a major link between the abstractions of mathematics and the real world in order to describe and compare objects and data.

Students:

- **derive and apply formulas to find measures such as length, area, volume, weight, time, and angle in real-world contexts.**
- **choose the appropriate tools for measurement.**
- **use dimensional analysis techniques.**
- **use statistical methods including measures of central tendency to describe and compare data.**
- **use trigonometry as a method to measure indirectly.**
- **apply proportions to scale drawings, computer-assisted design blueprints, and direct variation in order to compute indirect measurements.**
- **relate absolute value, distance between two points, and the slope of a line to the coordinate plane.**
- **understand error in measurement and its consequence on subsequent calculations.**
- **use geometric relationships in relevant measurement problems involving geometric concepts.**

This is evident, for example, when students:

▲ change mph to ft/sec.
▲ use the tangent ratio to determine the height of a tree.
▲ determine the distance between two points in the coordinate plane.

Uncertainty

6. Students use ideas of uncertainty to illustrate that mathematics involves more than exactness when dealing with everyday situations.

Students:

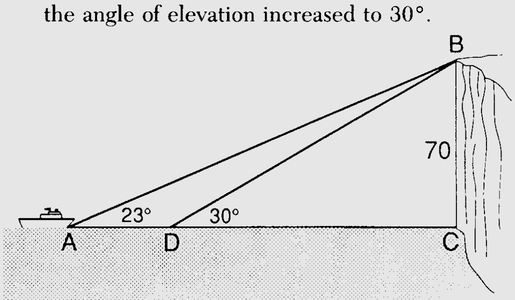
- **judge the reasonableness of results obtained from applications in algebra, geometry, trigonometry, probability, and statistics.**
- **judge the reasonableness of a graph produced by a calculator or computer.**
- **use experimental or theoretical probability to represent and solve problems involving uncertainty.**
- **use the concept of random variable in computing probabilities.**
- **determine probabilities using permutations and combinations.**

This is evident, for example, when students:

▲ construct a tree diagram or sample space for a compound event.
▲ calculate the probability of winning the New York State Lottery.
▲ develop simulations for probability problems for which they do not have theoretical solutions.

Sample Problems

39 As shown in the accompanying diagram, a ship is headed directly toward a coastline formed by a vertical cliff $\overline{BC}$, 70 meters high. At point *A*, the angle of elevation from the ship to *B*, the top of the cliff, is 23°. A few minutes later at point *D*, the angle of elevation increased to 30°.



a To the *nearest meter*, find:
(1) *DC* [3]
(2) *AC* [3]
(3) *AB* [3]

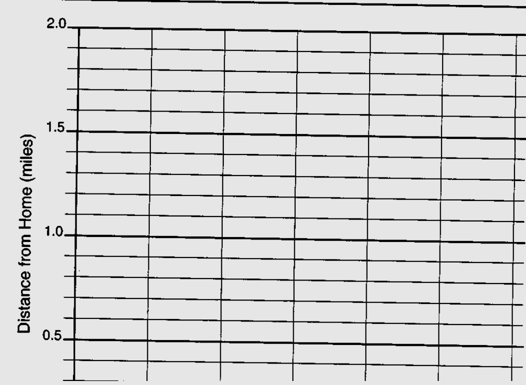
b To the *nearest meter*, what is the distance between the ship's position at the two sightings?

Every morning Walker Bryce walks 1.7 miles to school.

He leaves his house at 8:05 and walks 1.2 miles, then he waits for Bobby and Denise.

When they show up, all three of them start walking to school together. They arri[ve] ten minutes later at 8:55.

Draw a graph that could show Walker's journey to school.



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Patterns/Functions

7. Students use patterns and functions to develop mathematical power, appreciate the true beauty of mathematics, and construct generalizations that describe patterns simply and efficiently.

Students:

- **use function vocabulary and notation.**
- **represent and analyze functions using verbal descriptions, tables, equations, and graphs.**
- **translate among the verbal descriptions, tables, equations and graphic forms of functions.**
- **analyze the effect of parametric changes on the graphs of functions.**
- **apply linear, exponential, and quadratic functions in the solution of problems.**
- **apply and interpret transformations to functions.**
- **model real-world situations with the appropriate function.**
- **apply axiomatic structure to algebra and geometry.**
- **use computers and graphing calculators to analyze mathematical phenomena.**

This is evident, for example, when students:

▲ determine, in more than one way, whether or not a specific relation is a function.

▲ explain the relationship between the roots of a quadratic equation and the intercepts of its corresponding graph.

▲ use transformations to determine the inverse of a function.

Sample Problem

Fibonacci Pattern

This is the Fibonacci sequence:

1, 1, 2, 3, 5, 8, 13, 21, …

Each number (starting with the "2") is the sum of the previous two. For example,

1 + 1 = 2 and 2 + 3 = 5.

The number that comes after 21, in the above sequence, is 34 because:

13 + 21 = 34.

Now look at the pattern of odd and even numbers in this sequence. If we replace each odd number with "O" and each even with "E," we get:

O, O, E, O, O, E, O, O, …

Only one of the following statements is correct.

Decide which one you think is correct and explain in detail your choice.

A. The pattern, O, O, E, does NOT repeat forever.

B. The pattern, O, O, E, repeats forever.

Standard 3—Mathematics

Four-year sequence in mathematics

Mathematical Reasoning

1. Students use mathematical reasoning to analyze mathematical situations, make conjectures, gather evidence, and construct an argument.

Students:

- **construct indirect proofs or proofs using mathematical induction.**
- **investigate and compare the axiomatic structures of various geometries.**

This is evident, for example, when students:

▲ prove indirectly that: if n^2 is even, n is even.

▲ prove using mathematical induction that: $1 + 3 + 5 + \ldots + (2n - 1) = n^2$.

▲ explain the axiomatic differences between plane and spherical geometries.

Number and Numeration

2. Students use number sense and numeration to develop an understanding of the multiple uses of numbers in the real world, the use of numbers to communicate mathematically, and the use of numbers in the development of mathematical ideas.

Students:

- **understand the concept of infinity.**
- **drecognize the hierarchy of the complex number system.**
- **model the structure of the complex number system.**
- **recognize when to use and how to apply the field properties.**

This is evident, for example, when students:

▲ relate the concept of infinity when graphing the tangent function.

▲ show that the set of complex numbers form a field under the operations of addition and multiplication.

▲ show that the set of complex numbers forms a field under the operations of addition and multiplication.

▲ represent a complex number in polar form.

Key ideas are identified by numbers (1).
Performance indicators are identified by bullets (•).
Sample tasks are identified by triangles (▲).

Students will understand mathematics and become mathematically confident by communicating and reasoning mathematically, by applying mathematics in real-world settings, and by solving problems through the integrated study of number systems, geometry, algebra, data analysis, probability, and trigonometry.

Operations

3. Students use mathematical operations and relationships among them to understand mathematics.

Students:

- **use appropriate techniques, including graphing utilities, to perform basic operations on matrices.**
- **use rational exponents on real numbers and all operations on complex numbers.**
- **combine functions using the basic operations and the composition of two functions.**

This is evident, for example, when students:

▲ relate specific matrices to certain types of transformations of points on the coordinate plane.

▲ evaluate expressions with fractional exponents, such as $8^{2/3}\,4^{-1/2}$.

▲ determine the value of compound functions such as (f o g) (x).

Modeling/Multiple Representation

4. Students use mathematical modeling/multiple representation to provide a means of presenting, interpreting, communicating, and connecting mathematical information and relationships.

Students:

- **model vector quantities both algebraically and geometrically.**
- **represent graphically the sum and difference of two complex numbers.**
- **model and solve problems that involve absolute value, vectors, and matrices.**
- **model quadratic inequalities both algebraically and graphically.**
- **model the composition of transformations.**
- **determine the effects of changing parameters of the graphs of functions.**
- **use polynomial, rational, trigonometric, and exponential functions to model real-world relationships.**
- **use algebraic relationships to analyze the conic sections.**
- **use circular functions to study and model periodic real-world phenomena.**
- **illustrate spatial relationships using perspective, projections, and maps.**
- **represent problem situations using discrete structures such as finite graphs, matrices, sequences, and recurrence relations.**
- **analyze spatial relationships using the Cartesian coordinate system in three dimensions.**

This is evident, for example, when students:

▲ determine coordinates which lie in the solution of the quadriatic inequality, such as $y < x^2 + 4x + 2$.

▲ find the distance between two points in a three-dimension coordinate system.

▲ describe what happens to the graph when b increases in the function $y = x^2 + bx + c$.

Standard 3—Mathematics

Four-year sequence in mathematics

Measurement

5. Students use measurement in both metric and English measure to provide a major link between the abstractions of mathematics and the real world in order to describe and compare objects and data.

Students:

- **derive and apply formulas relating angle measure and arc degree measure in a circle.**
- **prove and apply theorems related to lengths of segments in a circle.**
- **define the trigonometric functions in terms of the unit circle.**
- **relate trigonometric relationships to the area of a triangle and to the general solutions of triangles.**
- **apply the normal curve and its properties to familiar contexts.**
- **design a statistical experiment to study a problem and communicate the outcomes, including dispersion.**
- **use statistical methods, including scatter plots and lines of best fit, to make predictions.**
- **apply the conceptual foundation of limits, infinite sequences and series, the area under a curve, rate of change, inverse variation, and the slope of a tangent line to authentic problems in mathematics and other disciplines.**
- **determine optimization points on a graph.**
- **use derivatives to find maximum, minimum, and inflection points of a function.**

This is evident, for example, when students:

▲ use a chi-square test to determine if one cola really tastes better than another cola.

▲ can illustrate the various line segments which represent the sine, cosine, and tangent of a given angle on the unit circle.

▲ calculate the first derivative of a function using the limit definition.

Uncertainty

6. Students use ideas of uncertainty to illustrate that mathematics involves more than exactness when dealing with everyday situations.

Students:

- **interpret probabilities in real-world situations.**
- **use a Bernoulli experiment to determine probabilities for experiments with exactly two outcomes.**
- **use curve fitting to predict from data.**
- **apply the concept of random variable to generate and interpret probability distributions.**
- **create and interpret applications of discrete and continuous probability distributions.**
- **make predictions based on interpolations and extrapolations from data.**
- **obtain confidence intervals and test hypotheses using appropriate statistical methods.**
- **approximate the roots of polynomial equations.**

This is evident, for example, when students:

▲ verify the probabilities listed for the state lottery for second, third, and fourth prize.

▲ use graphing calculators to generate a curve of best fit for an array of data using linear regression.

▲ determine the probability of getting at least 3 heads on 6 flips of a fair coin.

Key ideas are identified by numbers (1).
Performance indicators are identified by bullets (•).
Sample tasks are identified by triangles (▲).

Students will understand mathematics and become mathematically confident by communicating and reasoning mathematically, by applying mathematics in real-world settings, and by solving problems through the integrated study of number systems, geometry, algebra, data analysis, probability, and trigonometry.

Patterns/Functions

7. Students use patterns and functions to develop mathematical power, appreciate the true beauty of mathematics, and construct generalizations that describe patterns simply and efficiently.

Students:

- **solve equations with complex roots using a variety of algebraic and graphical methods with appropriate tools.**
- **understand and apply the relationship between the rectangular form and the polar form of a complex number.**
- **evaluate and form the composition of functions.**
- **use the definition of a derivative to examine the properties of a function.**
- **solve equations involving fractions, absolute values, and radicals.**
- **use basic transformations to demonstrate similarity and congruence of figures.**
- **identify and differentiate between direct and indirect isometries.**
- **analyze inverse functions using transformations.**
- **apply the ideas of symmetries in sketching and analyzing graphs of functions.**
- **use the normal curve to answer questions about data.**
- **develop methods to solve trigonometric equations and verify trigonometric functions.**
- **describe patterns produced by processes of geometric change, formally connecting iteration, approximations, limits, and fractals.**
- **extend patterns and compute the nth term in numerical and geometric sequences.**
- **use the limiting process to analyze infinite sequences and series.**
- **use algebraic and geometric iteration to explore patterns and solve problems.**
- **solve optimization problems.**
- **use linear programming and difference equations in the solution of problems.**

This is evident, for example, when students:

▲ transform polar coordinates into rectangular forms.
▲ find the maximum height of an object projects upward with a given initial velocity.
▲ find the limit of expressions like $\frac{n - 2}{3n + 5}$ as n goes to infinity.