InspireData™ Standards Match

MICHIGAN

Mathematics Curriculum Framework

Meeting curriculum standards is a major focus in education today. This document highlights the correlation of **InspireData™** with the **Michigan Mathematics Curriculum Framework.**

The Inspired Standards Match is designed to demonstrate the many ways InspireData supports the standards and to give educators ideas for using this tool to meet learning goals.

How to read the InspireData Standards Match:

- **Yellow** highlight indicates a standard or objective that can be supported by the use of InspireData databases, database templates, user generated databases, lesson plans or program features.
- **Green** notes list details about how InspireData can be used to meet the standards, including examples of specific databases, lesson plans or features that support them.

Thank you for your interest in InspireData!

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FOURTH GRADE MATHEMATICS

GRADE LEVEL CONTENT EXPECTATIONS

4 MATH

v.12.05

NUMBER & OPERATIONS

ALGEBRA

MEASUREMENT

GEOMETRY

DATA & PROBABILITY

Welcome to Michigan's K-8 Grade Level Content Expectations

Purpose & Overview

In 2004, the Michigan Department of Education embraced the challenge of creating Grade Level Content Expectations in response to the federal No Child Left Behind Act of 2001. This act mandated the existence of a set of comprehensive state grade level assessments that are designed based on rigorous grade level content.

In this global economy, it is essential that Michigan students possess personal, social, occupational, civic, and quantitative literacy. Mastery of the knowledge and essential skills defined in Michigan's Grade Level Content Expectations will increase students' ability to be successful academically, contribute to the future businesses that employ them and the communities in which they choose to live.

The Grade Level Content Expectations build from the Michigan Curriculum Framework and its Teaching and Assessment Standards. Reflecting best practices and current research, they provide a set of clear and rigorous expectations for all students and provide teachers with clearly defined statements of what students should know and be able to do as they progress through school.

Why Create a 12.05 Version of the Expectations?

The Office of School Improvement is committed to creating the best possible product for educators. This commitment served as the impetus for the revision of the 6.04 edition that was previously released in June of 2004. This new version, v.12.05, refines and clarifies the original expectations, while preserving their essence and original intent. As education continues to evolve, it is important to remember that each curriculum document should be considered as a work in progress, and will continue to be refined to improve the quality.

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MICHIGAN Department of Education
Office of School Improvement

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Curriculum

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The expectations in this document are divided into strands with multiple domains within each, as shown below. The skills and content addressed in these expectations will in practice be woven together into a coherent, Mathematics curriculum. The domains in each mathematics strand are broader, more conceptual groupings. In several of the strands, the "domains" are similar to the "standards" in Principles and Standards for School Mathematics from the National Council of Teachers of Mathematics.

To allow for ease in referencing expectations, each expectation has been coded with a strand, domain, grade-level, and expectation number. For example, **M.UN.00.01** indicates:

M - Measurement strand

UN - Units & systems of measurement domain of the Measurement strand

00 - Kindergarten Expectation

01 - First Expectation in the Grade-Level view of the Measurement strand

Strand 1 Number & Operations	**Strand 2 Algebra**	**Strand 3 Measurement**	**Strand 4 Geometry**	**Strand 5 Data and Probability**
Domains				
Meaning, notation, place value, and comparisons (ME) Number relationships and meaning of operations (MR) Fluency with operations and estimation (FL)	Patterns, relations, functions, and change (PA) Representation (RP) Formulas, expressions, equations, and inequalities (RP)	Units and systems of measurement (UN) Techniques and formulas for measurement (TE) Problem solving involving measurement (PS)	Geometric shape, properties, and mathematical arguments (GS) Location and spatial relationships (LO) Spatial reasoning and geometric modeling (SR) Transformation and symmetry (TR)	Data representation (RE) Data interpretation and analysis (AN) Probability (PR)

Preparing Students for Academic Success

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NUMBER AND OPERATIONS

Understand and use number notation and place value

N.ME.04.01 Read and write numbers to 1,000,000; relate them to the quantities they represent; compare and order.

N.ME.04.02 Compose and decompose numbers using place value to 1,000,000's, e.g., 25,068 is 2 ten thousands, 5 thousands, 0 hundreds, 6 tens, and 8 ones.

N.ME.04.03 Understand the magnitude of numbers up to 1,000,000; recognize the place values of numbers and the relationship of each place value to the place to its right, e.g., 1,000 is 10 hundreds.

Use factors and multiples

N.ME.04.04 *Find all factors of any whole number through 50, list factor pairs, and determine if a one-digit number is a factor of a given whole number.**

N.ME.04.05 *List the first ten multiples of a given one-digit whole number; determine if a whole number is a multiple of a given one-digit whole number.**

N.MR.04.06 Know that some numbers including 2, 3, 5, 7, and 11 have exactly two factors (1 and the number itself) and are called prime numbers.

N.MR.04.07 *Use factors and multiples to compose and decompose whole numbers.**

Add and subtract whole numbers

N.FL.04.08 Add and subtract whole numbers fluently.

Multiply and divide whole numbers

N.ME.04.09 Multiply two-digit numbers by 2, 3, 4, and 5 using the distributive property, e.g., $21 \times 3 = (1 + 20) \times 3 = (1 \times 3) + (20 \times 3) = 3 + 60 = 63$.

N.FL.04.10 Multiply fluently any whole number by a one-digit number and a three-digit number by a two-digit number; for a two-digit by one-digit multiplication use distributive property to develop meaning for the algorithm.

N.FL.04.11 Divide numbers up to four-digits by one-digit numbers and by 10.

N.FL.04.12 *Find the value of the unknowns in equations such as $a \div 10 = 25$; $125 \div b = 25$.**

N.MR.04.13 Use the relationship between multiplication and division to simplify computations and check results.

N.MR.04.14 *Solve contextual problems involving whole number multiplication and division.**

Read, interpret and compare decimal fractions

N.ME.04.15 Read and interpret decimals up to two decimal places; relate to money and place value decomposition.

N.ME.04.16 Know that terminating decimals represents fractions whose denominators are 10, 10×10, $10 \times 10 \times 10$, etc., e.g., powers of 10.

N.ME.04.17 Locate tenths and hundredths on a number line.

N.ME.04.18 Read, write, interpret, and compare decimals up to two decimal places.

N.MR.04.19 Write tenths and hundredths in decimal and fraction forms, and know the decimal equivalents for halves and fourths.

** revised expectations in italics*

Understand fractions

N.ME.04.20 Understand fractions as parts of a set of objects.

N.MR.04.21 Explain why equivalent fractions are equal, using models such as fraction strips or the number line for fractions with denominators of 12 or less, or equal to 100.

N.MR.04.22 *Locate fractions with denominators of 12 or less on the number line; include mixed numbers.**

N.MR.04.23 Understand the relationships among halves, fourths, and eighths and among thirds, sixths, and twelfths.

N.ME.04.24 *Know that fractions of the form $\frac{m}{n}$ where m is greater than n, are greater than 1 and are called improper fractions; locate improper fractions on the number line.**

N.MR.04.25 Write improper fractions as mixed numbers, and understand that a mixed number represents the number of "wholes" and the part of a whole remaining, e.g., $\frac{5}{4} = 1 + \frac{1}{4} = 1\frac{1}{4}$.

N.MR.04.26 Compare and order up to three fractions with denominators 2, 4, and 8, and 3, 6, and 12, including improper fractions and mixed numbers.

Add and subtract fractions

N.MR.04.27 *Add and subtract fractions less than 1 with denominators through 12 and/or 100, in cases where the denominators are equal or when one denominator is a multiple of the other,* e.g., $\frac{1}{12} + \frac{5}{12} = \frac{6}{12}$; $\frac{1}{6} + \frac{5}{12} = \frac{7}{12}$; $\frac{3}{10} - \frac{23}{100} = \frac{7}{100}$. *

N.MR.04.28 *Solve contextual problems involving sums and differences for fractions where one denominator is a multiple of the other (denominators 2 through 12, and 100).**

N.MR.04.29 *Find the value of an unknown in equations such as* $\frac{1}{8} + x = \frac{5}{8}$ or $\frac{3}{4} - y = \frac{1}{2}$.*

Multiply fractions by whole numbers

N.MR.04.30 Multiply fractions by whole numbers, using repeated addition and area or array models.

Add and subtract decimal fractions

N.MR.04.31 *For problems that use addition and subtraction of decimals through hundredths, represent with mathematical statements and solve.**

N.FL.04.32 *Add and subtract decimals through hundredths.**

Multiply and divide decimal fractions

N.FL.04.33 Multiply and divide decimals up to two decimal places by a one-digit whole number where the result is a terminating decimal, e.g., $0.42 \div 3 = 0.14$, but not $5 \div 3 = 1.\overline{6}$.

Estimate

N.FL.04.34 Estimate the answers to calculations involving addition, subtraction, or multiplication.

N.FL.04.35 Know when approximation is appropriate and use it to check the reasonableness of answers; be familiar with common place-value errors in calculations.

N.FL.04.36 Make appropriate estimations and calculations fluently with whole numbers using mental math strategies.

** revised expectations in italics*

MEASUREMENT

Measure using common tools and appropriate units

M.UN.04.01 Measure using common tools and select appropriate units of measure.

M.PS.04.02 Give answers to a reasonable degree of precision in the context of a given problem.

M.UN.04.03 Measure and compare integer temperatures in degrees.

M.TE.04.04 Measure surface area of cubes and rectangular prisms by covering and counting area of the faces.

Convert measurement units

M.TE.04.05 Carry out the following conversions from one unit of measure to a larger or smaller unit of measure: meters to centimeters, kilograms to grams, liters to milliliters, hours to minutes, minutes to seconds, years to months, weeks to days, feet to inches, ounces to pounds (using numbers that involve only simple calculations).

Use perimeter and area formulas

M.TE.04.06 Know and understand the formulas for perimeter and area of a square and a rectangle; calculate the perimeters and areas of these shapes and combinations of these shapes using the formulas.

M.TE.04.07 Find one dimension of a rectangle given the other dimension and its perimeter or area.

M.TE.04.08 Find the side of a square given its perimeter or area.

M.PS.04.09 Solve contextual problems about perimeter and area of squares and rectangles in compound shapes.

Understand right angles

M.TE.04.10 Identify right angles and compare angles to right angles.

Problem-solving

M.PS.04.11 Solve contextual problems about surface area.

GEOMETRY

Understand perpendicular, parallel, and intersecting lines

G.GS.04.01 Identify and draw perpendicular, parallel, and intersecting lines using a ruler and a tool or object with a square (90°) corner.

Identify basic geometric shapes and their components, and solve problems

G.GS.04.02 Identify basic geometric shapes including isosceles, equilateral, and right triangles, and use their properties to solve problems.

G.SR.04.03 Identify and count the faces, edges, and vertices of basic three-dimensional geometric solids including cubes, rectangular prisms, and pyramids; describe the shape of their faces.

GEOMETRY

Recognize symmetry and transformations

G.TR.04.04 Recognize plane figures that have line symmetry.

G.TR.04.05 Recognize rigid motion transformations (flips, slides, turns) of a two-dimensional object.

DATA AND PROBABILITY

Represent and solve problems for given data

D.RE.04.01 Construct tables and bar graphs from given data.

D.RE.04.02 Order a given set of data, find the median, and specify the range of values.

D.RE.04.03 Solve problems using data presented in tables and bar graphs, e.g., compare data represented in two bar graphs and read bar graphs showing two data sets.

FIFTH GRADE MATHEMATICS

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5 MATH

v.12.05

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NUMBER AND OPERATIONS

Understand division of whole numbers

N.MR.05.01 Understand the meaning of division of whole numbers with and without remainders; relate division to fractions and to repeated subtraction.

N.MR.05.02 Relate division of whole numbers with remainders to the form $a = bq + r$, e.g., $34 \div 5 = 6$ r 4, so $5 \bullet 6 + 4 = 34$; note remainder (4) is less than divisor (5).

N.MR.05.03 Write mathematical statements involving division for given situations.

Multiply and divide whole numbers

N.FL.05.04 Multiply a multi-digit number by a two-digit number; recognize and be able to explain common computational errors such as not accounting for place value.

N.FL.05.05 *Solve applied problems involving multiplication and division of whole numbers.**

N.FL.05.06 Divide fluently up to a four-digit number by a two-digit number.

Find prime factorizations of whole numbers

N.MR.05.07 *Find the prime factorization of numbers from 2 through 50, express in exponential notation, e.g., $24 = 2^3 \times 3^1$, and understand that every whole number greater than 1 is either prime or can be expressed as a product of primes.**

Understand meaning of decimal fractions and percentages

N.ME.05.08 Understand the relative magnitude of ones, tenths, and hundredths and the relationship of each place value to the place to its right, e.g., one is 10 tenths, one tenth is 10 hundredths.

N.ME.05.09 Understand percentages as parts out of 100, use % notation, and express a part of a whole as a percentage.

Understand fractions as division statements; find equivalent fractions

N.ME.05.10 Understand a fraction as a statement of division, e.g., $2 \div 3 = \frac{2}{3}$, using simple fractions and pictures to represent.

N.ME.05.11 *Given two fractions, e.g., $\frac{1}{2}$ and $\frac{1}{4}$, express them as fractions with a common denominator, but not necessarily a least common denominator, e.g., $\frac{1}{2} = \frac{4}{8}$ and $\frac{3}{4} = \frac{6}{8}$; use denominators less than 12 or factors of 100.**

Multiply and divide fractions

N.ME.05.12 *Find the product of two unit fractions with small denominators using an area model.**

N.MR.05.13 *Divide a fraction by a whole number and a whole number by a fraction, using simple unit fractions.**

Add and subtract fractions using common denominators

N.FL.05.14 *Add and subtract fractions with unlike denominators through 12 and/or 100, using the common denominator that is the product of the denominators of the 2 fractions, e.g., $\frac{3}{8} + \frac{7}{10}$: use 80 as the common denominator.**

* *revised expectations in italics*

Multiply and divide by powers of ten

N.MR.05.15 Multiply a whole number by powers of 10: 0.01, 0.1, 1, 10, 100, 1,000; and identify patterns.

N.FL.05.16 Divide numbers by 10's, 100's, 1,000's using mental strategies.

N.MR.05.17 Multiply one-digit and two-digit whole numbers by decimals up to two decimal places.

Solve applied problems with fractions

N.FL.05.18 *Use mathematical statements to represent an applied situation involving addition and subtraction of fractions.**

N.MR.05.19 *Solve contextual problems that involve finding sums and differences of fractions with unlike denominators using knowledge of equivalent fractions.**

N.FL.05.20 *Solve applied problems involving fractions and decimals; include rounding of answers and checking reasonableness.**

N.MR.05.21 *Solve for the unknown in equations such as* $\frac{1}{4} + x = \frac{7}{12}$.*

Express, interpret, and use ratios; find equivalences

N.MR.05.22 Express fractions and decimals as percentages and vice versa.

N.ME.05.23 Express ratios in several ways given applied situations, e.g., 3 cups to 5 people, 3 : 5, $\frac{3}{5}$; recognize and find equivalent ratios.

MEASUREMENT

Know, and convert among, measurement units within a given system

M.UN.05.01 Recognize the equivalence of 1 liter, 1,000 ml and 1,000 cm^3 and include conversions among liters, milliliters, and cubic centimeters.

M.UN.05.02 Know the units of measure of volume: cubic centimeter, cubic meter, cubic inches, cubic feet, cubic yards, and use their abbreviations (cm^3, m^3, in^3, ft^3, yd^3).

M.UN.05.03 Compare the relative sizes of one cubic inch to one cubic foot, and one cubic centimeter to one cubic meter.

M.UN.05.04 Convert measurements of length, weight, area, volume, and time within a given system using easily manipulated numbers.

Find areas of geometric shapes using formulas

M.PS.05.05 Represent relationships between areas of rectangles, triangles, and parallelograms using models.

M.TE.05.06 Understand and know how to use the area formula of a triangle:

$A = \frac{1}{2}bh$ (where b is length of the base and h is the height), and represent using models and manipulatives.

M.TE.05.07 Understand and know how to use the area formula for a parallelogram:

$A = bh$, and represent using models and manipulatives.

Understand the concept of volume

M.TE.05.08 Build solids with unit cubes and state their volumes.

M.TE.05.09 Use filling (unit cubes or liquid), and counting or measuring to find the volume of a cube and rectangular prism.

M.PS.05.10 Solve applied problems about the volumes of rectangular prisms using multiplication and division and using the appropriate units.

* *revised expectations in italics*

GEOMETRY

Know the meaning of angles, and solve problems

G.TR.05.01 Associate an angle with a certain amount of turning; know that angles are measured in degrees; understand that 90°, 180°, 270°, and 360° are associated respectively, with $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{3}{4}$, and full turns.

G.GS.05.02 Measure angles with a protractor and classify them as acute, right, obtuse, or straight.

G.GS.05.03 Identify and name angles on a straight line and vertical angles.

G.GS.05.04 Find unknown angles in problems involving angles on a straight line, angles surrounding a point, and vertical angles.

G.GS.05.05 Know that angles on a straight line add up to 180° and angles surrounding a point add up to 360°; justify informally by "surrounding" a point with angles.

G.GS.05.06 Understand why the sum of the interior angles of a triangle is 180° and the sum of the interior angles of a quadrilateral is 360°, and use these properties to solve problems.

Solve problems about geometric shapes

G.GS.05.07 Find unknown angles and sides using the properties of: triangles, including right, isosceles, and equilateral triangles; parallelograms, including rectangles and rhombuses; and trapezoids.

DATA AND PROBABILITY

Construct and interpret line graphs

D.RE.05.01 Read and interpret line graphs, and solve problems based on line graphs, e.g., distance-time graphs, and problems with two or three line graphs on same axes, comparing different data.

D.RE.05.02 Construct line graphs from tables of data; include axis labels and scale.

Find and interpret mean and mode for a given set of data

D.AN.05.03 Given a set of data, find and interpret the mean (using the concept of fair share) and mode.

D.AN.05.04 Solve multi-step problems involving means.

SIXTH GRADE MATHEMATICS

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NUMBER AND OPERATIONS

Multiply and divide fractions

N.MR.06.01 Understand division of fractions as the inverse of multiplication, e.g., if $\frac{4}{5} \div \frac{2}{3} = \square$, then $\frac{2}{3} \bullet \square = \frac{4}{5}$, so $\square = \frac{4}{5} \bullet \frac{3}{2} = \frac{12}{10}$.

N.FL.06.02 Given an applied situation involving dividing fractions, write a mathematical statement to represent the situation.

N.MR.06.03 Solve for the unknown in equations such as $\frac{1}{4} \div \square = 1$, $\frac{3}{4} \div \square = \frac{1}{4}$, and $\frac{1}{2} = 1 \bullet \square$.

N.FL.06.04 Multiply and divide any two fractions, including mixed numbers, fluently.

Represent rational numbers as fractions or decimals

N.ME.06.05 Order rational numbers and place them on the number line.

N.ME.06.06 Represent rational numbers as fractions or terminating decimals when possible, and translate between these representations.

N.ME.06.07 Understand that a fraction or a negative fraction is a quotient of two integers, e.g., $-\frac{8}{3}$ is -8 divided by 3.

Add and subtract integers and rational numbers

N.MR.06.08 *Understand integer subtraction as the inverse of integer addition. Understand integer division as the inverse of integer multiplication.**

N.FL.06.09 *Add and multiply integers between -10 and 10; subtract and divide integers using the related facts. Use the number line and chip models for addition and subtraction.**

N.FL.06.10 Add, subtract, multiply and divide positive rational numbers fluently.

Find equivalent ratios

N.ME.06.11 Find equivalent ratios by scaling up or scaling down.

Solve decimal, percentage and rational number problems

N.FL.06.12 Calculate part of a number given the percentage and the number.

N.MR.06.13 *Solve contextual problems involving percentages such as sales taxes and tips.**

N.FL.06.14 For applied situations, estimate the answers to calculations involving operations with rational numbers.

N.FL.06.15 Solve applied problems that use the four operations with appropriate decimal numbers.

Use exponents

N.ME.06.16 *Understand and use integer exponents, excluding powers of negative bases; express numbers in scientific notation.**

Understand rational numbers and their location on the number line

N.ME.06.17 Locate negative rational numbers (including integers) on the number line; know that numbers and their negatives add to 0, and are on opposite sides and at equal distance from 0 on a number line.

N.ME.06.18 Understand that rational numbers are quotients of integers (non zero denominators), e.g., a rational number is either a fraction or a negative fraction.

N.ME.06.19 Understand that 0 is an integer that is neither negative nor positive.

N.ME.06.20 Know that the absolute value of a number is the value of the number ignoring the sign; or is the distance of the number from 0.

** revised expectations in italics*

ALGEBRA

Calculate rates

A.PA.06.01 Solve applied problems involving rates, including speed, e.g., if a car is going 50 mph, how far will it go in $3\frac{1}{2}$ hours?

Understand the coordinate plane

A.RP.06.02 Plot ordered pairs of integers and use ordered pairs of integers to identify points in all four quadrants of the coordinate plane.

Use variables, write expressions and equations, and combine like terms

A.FO.06.03 Use letters, with units, to represent quantities in a variety of contexts, e.g., y lbs., k minutes, x cookies.

A.FO.06.04 Distinguish between an algebraic expression and an equation.

A.FO.06.05 Use standard conventions for writing algebraic expressions, e.g., $2x + 1$ means "two times x, plus 1" and $2(x + 1)$ means "two times the quantity $(x + 1)$."

A.FO.06.06 Represent information given in words using algebraic expressions and equations.

A.FO.06.07 Simplify expressions of the first degree by combining like terms, and evaluate using specific values.

Represent linear functions using tables, equations, and graphs

A.RP.06.08 Understand that relationships between quantities can be suggested by graphs and tables.

A.PA.06.09 *Solve problems involving linear functions whose input values are integers; write the equation; graph the resulting ordered pairs of integers, e.g., given c chairs, the "leg function" is 4c; if you have 5 chairs, how many legs?; if you have 12 legs, how many chairs?**

A.RP.06.10 Represent simple relationships between quantities using verbal descriptions, formulas or equations, tables, and graphs, e.g., perimeter-side relationship for a square, distance-time graphs, and conversions such as feet to inches.

Solve equations

A.FO.06.11 *Relate simple linear equations with integer coefficients, e.g., $3x = 8$ or $x + 5 = 10$, to particular contexts and solve.**

A.FO.06.12 Understand that adding or subtracting the same number to both sides of an equation creates a new equation that has the same solution.

A.FO.06.13 Understand that multiplying or dividing both sides of an equation by the same non-zero number creates a new equation that has the same solutions.

A.FO.06.14 Solve equations of the form $ax + b = c$, e.g., $3x + 8 = 15$ by hand for positive integer coefficients less than 20, use calculators otherwise, and interpret the results.

MEASUREMENT

Convert within measurement systems

M.UN.06.01 Convert between basic units of measurement within a single measurement system, e.g., square inches to square feet.

Find volume and surface area

M.PS.06.02 Draw patterns (of faces) for a cube and rectangular prism that, when cut, will cover the solid exactly (nets).

M.TE.06.03 Compute the volume and surface area of cubes and rectangular prisms given the lengths of their sides, using formulas.

* *revised expectations in italics*

GEOMETRY

Understand and apply basic properties

G.GS.06.01 Understand and apply basic properties of lines, angles, and triangles, including:

- triangle inequality
- relationships of vertical angles, complementary angles, supplementary angles
- congruence of corresponding and alternate interior angles when parallel lines are cut by a transversal, and that such congruencies imply parallel lines
- locate interior and exterior angles of any triangle, and use the property that an exterior angle of a triangle is equal to the sum of the remote (opposite) interior angles
- know that the sum of the exterior angles of a convex polygon is 360°.

Understand the concept of congruence and basic transformations

G.GS.06.02 Understand that for polygons, congruence means corresponding sides and angles have equal measures.

G.TR.06.03 Understand the basic rigid motions in the plane (reflections, rotations, translations), relate these to congruence, and apply them to solve problems.

G.TR.06.04 Understand and use simple compositions of basic rigid transformations, e.g., a translation followed by a reflection.

Construct geometric shapes

G.SR.06.05 Use paper folding to perform basic geometric constructions of perpendicular lines, midpoints of line segments and angle bisectors; justify informally.

DATA AND PROBABILITY

Understand the concept of probability and solve problems

D.PR.06.01 Express probabilities as fractions, decimals, or percentages between 0 and 1; know that 0 probability means an event will not occur and that probability 1 means an event will occur.

D.PR.06.02 Compute probabilities of events from simple experiments with equally likely outcomes, e.g., tossing dice, flipping coins, spinning spinners, by listing all possibilities and finding the fraction that meets given conditions.

SEVENTH GRADE MATHEMATICS

GRADE LEVEL CONTENT EXPECTATIONS

7 MATH

v12.05

NUMBER & OPERATIONS

ALGEBRA

MEASUREMENT

GEOMETRY

DATA & PROBABILITY

Welcome to Michigan's K-8 Grade Level Content Expectations

Purpose & Overview

In 2004, the Michigan Department of Education embraced the challenge of creating Grade Level Content Expectations in response to the federal No Child Left Behind Act of 2001. This act mandated the existence of a set of comprehensive state grade level assessments that are designed based on rigorous grade level content.

In this global economy, it is essential that Michigan students possess personal, social, occupational, civic, and quantitative literacy. Mastery of the knowledge and essential skills defined in Michigan's Grade Level Content Expectations will increase students' ability to be successful academically, contribute to the future businesses that employ them and the communities in which they choose to live.

The Grade Level Content Expectations build from the Michigan Curriculum Framework and its Teaching and Assessment Standards. Reflecting best practices and current research, they provide a set of clear and rigorous expectations for all students and provide teachers with clearly defined statements of what students should know and be able to do as they progress through school.

Why Create a 12.05 Version of the Expectations?

The Office of School Improvement is committed to creating the best possible product for educators. This commitment served as the impetus for the revision of the 6.04 edition that was previously released in June of 2004. This new version, v.12.05, refines and clarifies the original expectations, while preserving their essence and original intent. As education continues to evolve, it is important to remember that each curriculum document should be considered as a work in progress, and will continue to be refined to improve the quality.

The revision process greatly improved the continuity from one grade to the next, and better ensured coherence both in content and pedagogy. To obtain more specific details about the revisions, please refer to the addendum included in this document. The forward of the ***Across the Grades v.12.05*** companion document also clarifies the types of changes made. Educators can access the *Across the Grades* companion document by visiting the Michigan Department of Education Grade Level Content Expectations web page at www.michigan.gov/glce.

Assessment

The Grade Level Content Expectations document is intended to be a state assessment tool with the expectations written to convey expected performances by students. The Office of Assessment and Accountability was involved in the development of version 12.05 and has incorporated the changes in the construction of test and item specifications for the K-8 Michigan Education Assessment Program (MEAP) and MI-Access. This updated version will assist us in the creation of companion documents, content examples, and to guide program planners in focusing resources and energy.

MICHIGAN Department of Education
Office of School Improvement

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Curriculum

Using this document as a focal point in the school improvement process, schools and districts can generate conversations among stakeholders concerning current policies and practices to consider ways to improve and enhance student achievement. Together, stakeholders can use these expectations to guide curricular and instructional decisions, identify professional development needs, and assess student achievement.

Understanding the Organizational Structure

The expectations in this document are divided into strands with multiple domains within each, as shown below. The skills and content addressed in these expectations will in practice be woven together into a coherent, Mathematics curriculum. The domains in each mathematics strand are broader, more conceptual groupings. In several of the strands, the "domains" are similar to the "standards" in Principles and Standards for School Mathematics from the National Council of Teachers of Mathematics.

To allow for ease in referencing expectations, each expectation has been coded with a strand, domain, grade-level, and expectation number. For example, **M.UN.00.01** indicates:

M - Measurement strand

UN - Units & systems of measurement domain of the Measurement strand

00 - Kindergarten Expectation

01 - First Expectation in the Grade-Level view of the Measurement strand

Strand 1 Number & Operations	**Strand 2 Algebra**	**Strand 3 Measurement**	**Strand 4 Geometry**	**Strand 5 Data and Probability**
Domains				
Meaning, notation, place value, and comparisons (ME) Number relationships and meaning of operations (MR) Fluency with operations and estimation (FL)	Patterns, relations, functions, and change (PA) Representation (RP) Formulas, expressions, equations, and inequalities (RP)	Units and systems of measurement (UN) Techniques and formulas for measurement (TE) Problem solving involving measurement (PS)	Geometric shape, properties, and mathematical arguments (GS) Location and spatial relationships (LO) Spatial reasoning and geometric modeling (SR) Transformation and symmetry (TR)	Data representation (RE) Data interpretation and analysis (AN) Probability (PR)

Preparing Students for Academic Success

Within the hands of teachers, the Grade Level Content Expectations are converted into exciting and engaging learning for Michigan's students. As we use these expectations to develop units of instruction and plan instructional delivery, it is critical to keep in mind that content knowledge alone is not sufficient for academic success. Students must be able to apply knowledge in new situations, to solve problems by generating new ideas, and to make connections between what they learn in class to the world around them. The art of teaching is what makes the content of learning become a reality.

Through the collaborative efforts of Michigan educators and creation of professional learning communities, we can enable our young people to attain the highest standards, and thereby open doors for them to have fulfilling and successful lives.

NUMBER AND OPERATIONS

Understand derived quantities

N.MR.07.02 *Solve problems involving derived quantities such as density, velocity, and weighted averages.**

Understand and solve problems involving rates, ratios, and proportions

N.FL.07.03 Calculate rates of change including speed.

N.MR.07.04 Convert ratio quantities between different systems of units, such as feet per second to miles per hour.

N.FL.07.05 *Solve proportion problems using such methods as unit rate, scaling, finding equivalent fractions, and solving the proportion equation a/b = c/d; know how to see patterns about proportional situations in tables.**

Recognize irrational numbers

N.MR.07.06 Understand the concept of square root and cube root, and estimate using calculators.

Compute with rational numbers

N.FL.07.07 Solve problems involving operations with integers.

N.FL.07.08 *Add, subtract, multiply, and divide positive and negative rational numbers fluently.**

N.FL.07.09 Estimate results of computations with rational numbers.

** revised expectations in italics*

ALGEBRA

Understand and apply directly proportional relationships and relate to linear relationships

A.PA.07.01 *Recognize when information given in a table, graph, or formula suggests a directly proportional or linear relationship.**

A.RP.07.02 Represent directly proportional and linear relationships using verbal descriptions, tables, graphs, and formulas, and translate among these representations.

A.PA.07.03 *Given a directly proportional or other linear situation, graph and interpret the slope and intercept(s) in terms of the original situation; evaluate y = mx + b for specific x values, e.g., weight vs. volume of water, base cost plus cost per unit.**

A.PA.07.04 For directly proportional or linear situations, solve applied problems using graphs and equations, e.g., the heights and volume of a container with uniform cross-section; height of water in a tank being filled at a constant rate; degrees Celsius and degrees Fahrenheit; distance and time under constant speed.

A.PA.07.05 *Recognize and use directly proportional relationships of the form y = mx, and distinguish from linear relationships of the form y = mx + b, b non-zero; understand that in a directly proportional relationship between two quantities one quantity is a constant multiple of the other quantity.**

Understand and represent linear functions

A.PA.07.06 Calculate the slope from the graph of a linear function as the ratio of "rise/run" for a pair of points on the graph, and express the answer as a fraction and a decimal; understand that linear functions have slope that is a constant rate of change.

A.PA.07.07 Represent linear functions in the form y = x + b, y = mx, and y = mx + b, and graph, interpreting slope and y-intercept.

A.FO.07.08 *Find and interpret the x and/or y intercepts of a linear equation or function. Know that the solution to a linear equation of the form ax+b=0 corresponds to the point at which the graph of y=ax+b crosses the x axis.**

Understand and solve problems about inversely proportional relationships

A.PA.07.09 Recognize inversely proportional relationships in contextual situations; know that quantities are inversely proportional if their product is constant, e.g., the length and width of a rectangle with fixed area, and that an inversely proportional relationship is of the form y = k/x where k is some non-zero number.

A.RP.07.10 Know that the graph of y = k/x is not a line, know its shape, and know that it crosses neither the x nor the y-axis.

Apply basic properties of real numbers in algebraic contexts

A.PA.07.11 Understand and use basic properties of real numbers: additive and multiplicative identities, additive and multiplicative inverses, commutativity, associativity, and the distributive property of multiplication over addition.

Combine algebraic expressions and solve equations

A.FO.07.12 *Add, subtract, and multiply simple algebraic expressions of the first degree, e.g., (92x + 8y) – 5x + y, or x(x+2) and justify using properties of real numbers.**

A.FO.07.13 From applied situations, generate and solve linear equations of the form ax + b = c and ax + b = cx + d, and interpret solutions.

* *revised expectations in italics*

GEOMETRY

Draw and construct geometric objects

G.SR.07.01 Use a ruler and other tools to draw squares, rectangles, triangles, and parallelograms with specified dimensions.

G.SR.07.02 Use compass and straightedge to perform basic geometric constructions: the perpendicular bisector of a segment, an equilateral triangle, and the bisector of an angle; understand informal justifications.

Understand the concept of similar polygons, and solve related problems

G.TR.07.03 Understand that in similar polygons, corresponding angles are congruent and the ratios of corresponding sides are equal; understand the concepts of similar figures and scale factor.

G.TR.07.04 Solve problems about similar figures and scale drawings.

G.TR.07.05 Show that two triangles are similar using the criteria: corresponding angles are congruent (AAA similarity); the ratios of two pairs of corresponding sides are equal and the included angles are congruent (SAS similarity); ratios of all pairs of corresponding sides are equal (SSS similarity); use these criteria to solve problems and to justify arguments.

G.TR.07.06 Understand and use the fact that when two triangles are similar with scale factor of r, their areas are related by a factor of r^2.

DATA AND PROBABILITY

Represent and interpret data

D.RE.07.01 Represent and interpret data using circle graphs, stem and leaf plots, histograms, and box-and-whisker plots, and select appropriate representation to address specific questions.

D.AN.07.02 Create and interpret scatter plots and find line of best fit; use an estimated line of best fit to answer questions about the data.

Compute statistics about data sets

D.AN.07.03 Calculate and interpret relative frequencies and cumulative frequencies for given data sets.

D.AN.07.04 Find and interpret the median, quartiles, and interquartile range of a given set of data.

EIGHTH GRADE MATHEMATICS

GRADE LEVEL CONTENT EXPECTATIONS

8 MATH

v.12.05

NUMBER & OPERATIONS

ALGEBRA

MEASUREMENT

GEOMETRY

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NUMBER AND OPERATIONS

Understand real number concepts

N.ME.08.01 Understand the meaning of a square root of a number and its connection to the square whose area is the number; understand the meaning of a cube root and its connection to the volume of a cube.

N.ME.08.02 Understand meanings for zero and negative integer exponents.

N.ME.08.03 Understand that in decimal form, rational numbers either terminate or eventually repeat, and that calculators truncate or round repeating decimals; locate rational numbers on the number line; know fraction forms of common repeating decimals, e.g., $0.\overline{1}=\frac{1}{9}$; $0.\overline{3}=\frac{1}{3}$.

N.ME.08.04 Understand that irrational numbers are those that cannot be expressed as the quotient of two integers, and cannot be represented by terminating or repeating decimals; approximate the position of familiar irrational numbers, e.g., $\sqrt{2}$, $\sqrt{3}$, π, on the number line.

N.FL.08.05 Estimate and solve problems with square roots and cube roots using calculators.

N.FL.08.06 Find square roots of perfect squares and approximate the square roots of non-perfect squares by locating between consecutive integers, e.g., $\sqrt{130}$ is between 11 and 12.

Solve problems

N.MR.08.07 Understand percent increase and percent decrease in both sum and product form, e.g., 3% increase of a quantity x is x + .03x = 1.03x.

N.MR.08.08 Solve problems involving percent increases and decreases.

N.FL.08.09 Solve problems involving compounded interest or multiple discounts.

N.MR.08.10 Calculate weighted averages such as course grades, consumer price indices, and sports ratings.

N.FL.08.11 *Solve problems involving ratio units, such as miles per hour, dollars per pound, or persons per square mile.**

** revised expectations in italics*

ALGEBRA

Understand the concept of non-linear functions using basic examples

A.RP.08.01 *Identify and represent linear functions, quadratic functions, and other simple functions including inversely proportional relationships ($y = k/x$); cubics ($y = ax^3$); roots ($y = \sqrt{x}$); and exponentials ($y = a^x$, $a > 0$); using tables, graphs, and equations.**

A.PA.08.02 For basic functions, e.g., simple quadratics, direct and indirect variation, and population growth, describe how changes in one variable affect the others.

A.PA.08.03 Recognize basic functions in problem context, e.g., area of a circle is πr^2, volume of a sphere is $\frac{4}{3}\pi r^3$, and represent them using tables, graphs, and formulas.

A.RP.08.04 Use the vertical line test to determine if a graph represents a function in one variable.

Understand and represent quadratic functions

A.RP.08.05 Relate quadratic functions in factored form and vertex form to their graphs, and vice versa; in particular, note that solutions of a quadratic equation are the x-intercepts of the corresponding quadratic function.

A.RP.08.06 Graph factorable quadratic functions, finding where the graph intersects the x-axis and the coordinates of the vertex; use words "parabola" and "roots"; include functions in vertex form and those with leading coefficient –1, e.g., $y = x^2 - 36$, $y = (x - 2)^2 - 9$; $y = -x^2$; $y = -(x - 3)^2$.

Recognize, represent, and apply common formulas

A.FO.08.07 Recognize and apply the common formulas:

$(a + b)^2 = a^2 + 2ab + b^2$

$(a - b)^2 = a^2 - 2ab + b^2$

$(a + b)(a - b) = a^2 - b^2$; represent geometrically.

A.FO.08.08 Factor simple quadratic expressions with integer coefficients, e.g., $x^2 + 6x + 9$, $x^2 + 2x - 3$, and $x^2 - 4$; solve simple quadratic equations, e.g., $x^2 = 16$ or $x^2 = 5$ (by taking square roots); $x^2 - x - 6 = 0$, $x^2 - 2x = 15$ (by factoring); verify solutions by evaluation.

A.FO.08.09 Solve applied problems involving simple quadratic equations.

Understand solutions and solve equations, simultaneous equations, and linear inequalities

A.FO.08.10 Understand that to solve the equation $f(x) = g(x)$ means to find all values of x for which the equation is true, e.g., determine whether a given value, or values from a given set, is a solution of an equation (0 is a solution of $3x^2 + 2 = 4x + 2$, but 1 is not a solution).

A.FO.08.11 Solve simultaneous linear equations in two variables by graphing, by substitution, and by linear combination; estimate solutions using graphs; include examples with no solutions and infinitely many solutions.

A.FO.08.12 Solve linear inequalities in one and two variables, and graph the solution sets.

A.FO.08.13 Set up and solve applied problems involving simultaneous linear equations and linear inequalities.

* *revised expectations in italics*

GEOMETRY

Understand and use the Pythagorean Theorem

G.GS.08.01 Understand at least one proof of the Pythagorean Theorem; use the Pythagorean Theorem and its converse to solve applied problems including perimeter, area, and volume problems.

G.LO.08.02 Find the distance between two points on the coordinate plane using the distance formula; recognize that the distance formula is an application of the Pythagorean Theorem.

Solve problems about geometric figures

G.SR.08.03 Understand the definition of a circle; know and use the formulas for circumference and area of a circle to solve problems.

G.SR.08.04 Find area and perimeter of complex figures by sub-dividing them into basic shapes (quadrilaterals, triangles, circles).

G.SR.08.05 Solve applied problems involving areas of triangles, quadrilaterals, and circles.

Understand concepts of volume and surface area, and apply formulas

G.SR.08.06 Know the volume formulas for generalized cylinders ((area of base) × height), generalized cones and pyramids ($\frac{1}{3}$(area of base) × height), and spheres ($\frac{4}{3}\pi$ (radius)^3) and apply them to solve problems.

G.SR.08.07 Understand the concept of surface area, and find the surface area of prisms, cones, spheres, pyramids, and cylinders.

Visualize solids

G.SR.08.08 Sketch a variety of two-dimensional representations of three-dimensional solids including orthogonal views (top, front, and side), picture views (projective or isometric), and nets; use such two-dimensional representations to help solve problems.

Understand and apply concepts of transformation and symmetry

G.TR.08.09 Understand the definition of a dilation from a point in the plane, and relate it to the definition of similar polygons.

G.TR.08.10 Understand and use reflective and rotational symmetries of two-dimensional shapes and relate them to transformations to solve problems.

DATA AND PROBABILITY

Draw, explain, and justify conclusions based on data

D.AN.08.01 Determine which measure of central tendency (mean, median, mode) best represents a data set, e.g., salaries, home prices, for answering certain questions; justify the choice made.

D.AN.08.02 Recognize practices of collecting and displaying data that may bias the presentation or analysis.

Understand probability concepts for simple and compound events

D.PR.08.03 *Compute relative frequencies from a table of experimental results for a repeated event. Interpret the results using relationship of probability to relative frequency.**

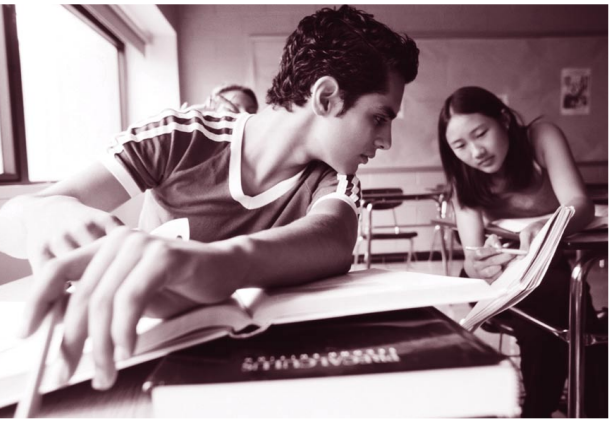
D.PR.08.04 Apply the Basic Counting Principle to find total number of outcomes possible for independent and dependent events, and calculate the probabilities using organized lists or tree diagrams.

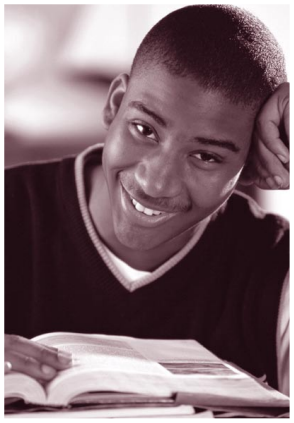
D.PR.08.05 *Find and/or compare the theoretical probability, the experimental probability, and/or the relative frequency of a given event.**

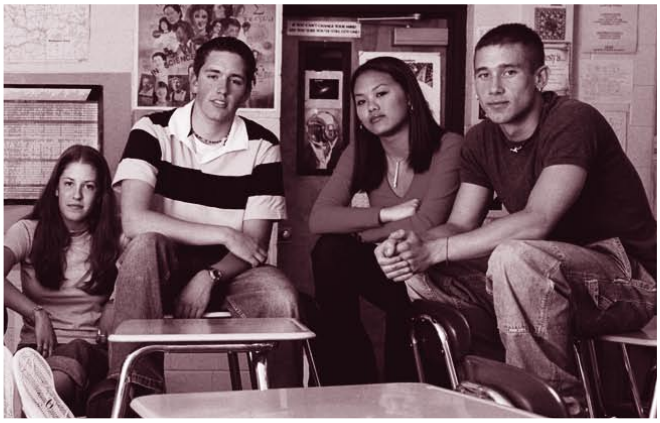
D.PR.08.06 Understand the difference between independent and dependent events, and recognize common misconceptions involving probability, e.g., Alice rolls a 6 on a die three times in a row; she is just as likely to roll a 6 on the fourth roll as she was on any previous roll.

* *revised expectations in italics*

High School Content Expectations







MATHEMATICS

- Quantitative Literacy and Logic
- Algebra and Functions
- Geometry and Trigonometry
- Statistics and Probability

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MICHIGAN Department of Education
5/06

Understanding the Organizational Structure

The expectations in this document are divided into four strands with multiple standards within each, as shown below. The skills and content addressed in these standards will, in practice, be woven together into a coherent, integrated Mathematics curriculum. The standards are comprehensive and are meant to be used as a guide to curriculum development.

STRAND 1 **Quantitative Literacy and Logic (L)**	**STRAND 2** **Algebra & Functions (A)**	**STRAND 3** **Geometry & Trigonometry (G)**	**STRAND 4** **Statistics & Probability (S)**
STANDARDS (and number of core expectations in each standard)			
L1: Reasoning About Numbers, Systems and Quantitative Situations (9) **L2: Calculation, Algorithms, and Estimation** (9) **L3: Measurement and Precision** (5) **L4: Mathematical Reasoning, Logic, and Proof** (10)	**A1: Expressions, Equations, and Inequalities** (16) **A2: Function** (39) **A3: Mathematical Modeling** (3)	**G1: Figures and Their Properties** (29) **G2: Relationships Between Figures** (10) **G3: Transformations of Figures in the Plane** (5)	**S1: Univariate Data—Examining Distributions** (9) **S2: Bivariate Data—Examining Relationships** (6) **S3: Samples, Surveys, and Experiments** (3) **S4: Probability Models and Probability Calculation** (4)
Recommended Quantitative Literacy and Logic Expectations (3)	*Recommended Algebra and Functions Expectations (5)*	*Recommended Geometry and Trigonometry Expectations (3)*	*Recommended Statistics and Probability Expectations (6)*

Core and Recommended Expectations

The expectations in this document represent what all Michigan high school graduates should know and be able to do in mathematics. With a focused and coherent set of **required core expectations**, teachers can provide both the breadth of mathematical experiences required for students to succeed in an increasingly competitive world economy, and also provide the depth required for mastery of fundamental mathematical ideas. There should be far less of the review and revisiting of topics that is typical in the high school mathematics curriculum. With a deep understanding of these expectations, students will make connections among fundamental mathematical ideas, and will be well-situated to use their mathematical knowledge and quantitative skills across the curriculum.

At the end of each strand, a set of **recommended expectations** is listed. These extensions represent content that is desirable and valuable for all students, but attention to these items should not displace or dilute the curricular emphasis of any of the core expectations. Teachers are encouraged to incorporate the recommended expectations into their instruction when their students have a solid foundation and are ready for enrichment or advanced learning. Recommended expectations may also be included in precalculus or statistics course content expectations when they are developed.

Addendum

The addendum represents additional mathematics recommended for all students. This section provides commentary on the importance of ensuring that all students have opportunities to learn precalculus and statistics and probability, as preparation for the workplace and/or for higher education. Specific course content expectations will be developed to address the themes and topics in the *Addendum*.

STRAND 1: QUANTITATIVE LITERACY AND LOGIC (L)

"In an increasingly complex world, adults are challenged to apply sophisticated quantitative knowledge and reasoning in their professional and personal lives. The technological demands of the workplace, the abundance of data in the political and public policy context, and the array of information involved in making personal and family decisions of all types necessitate an unprecedented facility not only with fundamental mathematical, statistical, and computing ideas and processes, but with higher-order abilities to apply and integrate those ideas and processes in a range of areas." [1]

The Michigan Grade Level Content Expectations in Mathematics for grades K-8 prescribe a thorough treatment of number, including strong emphasis on computational fluency and understanding of number concepts, to be completed largely by the sixth grade. The expectations in this Quantitative Literacy and Logic strand provide a definition of secondary school quantitative literacy for all students and emphasize the importance of logic as part of mathematics and in everyday life. They assume fluency (that is, efficiency and accuracy) in calculation with the basic number operations involving rational numbers in all forms (including percentages and decimals), without calculators.

Mathematical reasoning and logic are at the heart of the study of mathematics. As students progress through elementary and middle school, they increasingly are asked to explain and justify the thinking underlying their work. In high school, students peel away the contexts and study the language and thought patterns of formal mathematical reasoning. By learning logic and by constructing arguments and proofs, students will strengthen not only their knowledge and facility with mathematics, but also their ways of thinking in other areas of study and in their daily lives.

Connections and applications of number ideas and logic to other areas of mathematics, such as algebra, geometry, and statistics, are emphasized in this strand. Number representations and properties extend from the rational numbers into the real and complex numbers, as well as to other systems that students will encounter both in the workplace and in more advanced mathematics. The expectations for calculation, algorithms and estimation reflect important uses of number in a range of real-life situations. Ideas about measurement and precision tie closely to geometry.

[1] Estry, D., & Ferrini-Mundy, J. (January, 2005). Quantitative Literacy Task Force Final Report and Recommendations. East Lansing: Michigan State University.

STANDARD L1: REASONING ABOUT NUMBERS, SYSTEMS, AND QUANTITATIVE SITUATIONS

Based on their knowledge of the properties of arithmetic, students understand and reason about numbers, number systems, and the relationships between them. They represent quantitative relationships using mathematical symbols, and interpret relationships from those representations.

L1.1 Number Systems and Number Sense

L1.1.1 Know the different properties that hold in different number systems, and recognize that the applicable properties change in the transition from the positive integers, to all integers, to the rational numbers, and to the real numbers.

L1.1.2 Explain why the multiplicative inverse of a number has the same sign as the number, while the additive inverse of a number has the opposite sign.

L1.1.3 Explain how the properties of associativity, commutativity, and distributivity, as well as identity and inverse elements, are used in arithmetic and algebraic calculations.

L1.1.4 Describe the reasons for the different effects of multiplication by, or exponentiation of, a positive number by a number less than 0, a number between 0 and 1, and a number greater than 1.

L1.1.5 Justify numerical relationships (e.g., show that the sum of even integers is even; that every integer can be written as $3m+k$, where k is 0, 1, or 2, and m is an integer; or that the sum of the first n positive integers is $n(n+1)/2$).

L1.1.6 Explain the importance of the irrational numbers $\sqrt{2}$ and $\sqrt{3}$ in basic right triangle trigonometry; the importance of π because of its role in circle relationships; and the role of e in applications such as continuously compounded interest.

L1.2 Representations and Relationships

L1.2.1 Use mathematical symbols (e.g., interval notation, set notation, summation notation) to represent quantitative relationships and situations.

L1.2.2 Interpret representations that reflect absolute value relationships (e.g. $|x - a| \leq b$, *or* $a \pm b$) in such contexts as error tolerance.

L1.2.3 Use vectors to represent quantities that have magnitude and direction; interpret direction and magnitude of a vector numerically, and calculate the sum and difference of two vectors.

L1.2.4 Organize and summarize a data set in a table, plot, chart, or spreadsheet; find patterns in a display of data; understand and critique data displays in the media.

L1.3 Counting and Probabilistic Reasoning

L1.3.1 Describe, explain, and apply various counting techniques (e.g., finding the number of different 4-letter passwords; permutations; and combinations); relate combinations to Pascal's triangle; know when to use each technique.

L1.3.2 Define and interpret commonly used expressions of probability (e.g., chances of an event, likelihood, odds).

L1.3.3 Recognize and explain common probability misconceptions such as "hot streaks" and "being due."

STANDARD L2: CALCULATION, ALGORITHMS, AND ESTIMATION

Students calculate fluently, estimate proficiently, and describe and use algorithms in appropriate situations (e.g., approximating solutions to equations.) They understand the basic ideas of iteration and algorithms.

L2.1 Calculation Using Real and Complex Numbers

L2.1.1 Explain the meaning and uses of weighted averages (e.g., GNP, consumer price index, grade point average).

L2.1.2 Calculate fluently with numerical expressions involving exponents; use the rules of exponents; evaluate numerical expressions involving rational and negative exponents; transition easily between roots and exponents.

L2.1.3 Explain the exponential relationship between a number and its base 10 logarithm, and use it to relate rules of logarithms to those of exponents in expressions involving numbers.

L2.1.4 Know that the complex number i is one of two solutions to $x^2 = -1$.

L2.1.5 Add, subtract, and multiply complex numbers; use conjugates to simplify quotients of complex numbers.

L2.1.6 Recognize when exact answers aren't always possible or practical; use appropriate algorithms to approximate solutions to equations (e.g., to approximate square roots).

L2.2 Sequences and Iteration

L2.2.1 Find the nth term in arithmetic, geometric, or other simple sequences.

L2.2.2 Compute sums of finite arithmetic and geometric sequences.

L2.2.3 Use iterative processes in such examples as computing compound interest or applying approximation procedures.

STANDARD L3: MEASUREMENT AND PRECISION

Students apply measurement units and calculations, and understand the concept of error.

L3.1 Measurement Units, Calculations, and Scales

L3.1.1 Convert units of measurement within and between systems; explain how arithmetic operations on measurements affect units, and carry units through calculations correctly.

L3.1.2 Describe and interpret logarithmic relationships in such contexts as the Richter scale, the pH scale, or decibel measurements (e.g., explain why a small change in the scale can represent a large change in intensity); solve applied problems.

L3.2 Understanding Error

L3.2.1 Determine what degree of accuracy is reasonable for measurements in a given situation; express accuracy through use of significant digits, error tolerance, or percent of error; describe how errors in measurements are magnified by computation; recognize accumulated error in applied situations.

L3.2.2 Describe and explain round-off error, rounding, and truncating.

L3.2.3 Know the meaning of and interpret statistical significance, margin of error, and confidence level.

STANDARD L4: MATHEMATICAL REASONING, LOGIC, AND PROOF

Students understand mathematical reasoning as being grounded in logic and proof and can distinguish mathematical arguments from other types of arguments. They can interpret arguments made about quantitative situations in the popular media. Students know the language and laws of logic and can apply them in both mathematical and everyday settings. They write proofs using direct and indirect methods and use counterexamples appropriately to show that statements are false.

L4.1 Mathematical Reasoning

L4.1.1 Distinguish between inductive and deductive reasoning, identifying and providing examples of each.

L4.1.2 Differentiate between statistical arguments (statements verified empirically using examples or data) and logical arguments based on the rules of logic.

L4.1.3 Define and explain the roles of axioms (postulates), definitions, theorems, counterexamples, and proofs in the logical structure of mathematics; identify and give examples of each.

L4.2 Language and Laws of Logic

L4.2.1 Know and use the terms of basic logic (e.g., proposition, negation, truth and falsity, implication, if and only if, contrapositive, and converse).

L4.2.2 Use the connectives "NOT," "AND," "OR," and "IF…, THEN," in mathematical and everyday settings. Know the truth table of each connective and how to logically negate statements involving these connectives.

L4.2.3 Use the quantifiers "THERE EXISTS" and "ALL" in mathematical and everyday settings and know how to logically negate statements involving them.

L4.2.4 Write the converse, inverse, and contrapositive of an "If…, then…" statement; use the fact, in mathematical and everyday settings, that the contrapositive is logically equivalent to the original while the inverse and converse are not.

L4.3 Proof

L4.3.1 Know the basic structure for the proof of an "If…, then…" statement (assuming the hypothesis and ending with the conclusion) and know that proving the contrapositive is equivalent.

L4.3.2 Construct proofs by contradiction; use counterexamples, when appropriate, to disprove a statement.

L4.3.3 Explain the difference between a necessary and a sufficient condition within the statement of a theorem; determine the correct conclusions based on interpreting a theorem in which necessary or sufficient conditions in the theorem or hypothesis are satisfied.

RECOMMENDED:

***L1.2.5** Read and interpret representations from various technological sources, such as contour or isobar diagrams.

***L2.1.7** Understand the mathematical bases for the differences among voting procedures.

***L2.2.4** Compute sums of infinite geometric sequences.

STRAND 2: ALGEBRA AND FUNCTIONS (A)

In the middle grades, students see the progressive generalization of arithmetic to algebra. They learn symbolic manipulation skills and use them to solve equations. They study simple forms of elementary polynomial functions such as linear, quadratic, and power functions as represented by tables, graphs, symbols, and verbal descriptions.

In high school, students continue to develop their "symbol sense" by examining expressions, equations, and functions, and applying algebraic properties to solve equations. They construct a conceptual framework for analyzing any function and, using this framework, they revisit the functions they have studied before in greater depth. By the end of high school, their catalog of functions will encompass linear, quadratic, polynomial, rational, power, exponential, logarithmic, and trigonometric functions. They will be able to reason about functions and their properties and solve multi-step problems that involve both functions and equation-solving. Students will use deductive reasoning to justify algebraic processes as they solve equations and inequalities, as well as when transforming expressions.

This rich learning experience in Algebra will provide opportunities for students to understand both its structure and its applicability to solving real-world problems. Students will view algebra as a tool for analyzing and describing mathematical relationships, and for modeling problems that come from the workplace, the sciences, technology, engineering, and mathematics.

STANDARD A1: EXPRESSIONS, EQUATIONS, AND INEQUALITIES

Students recognize, construct, interpret, and evaluate expressions. They fluently transform symbolic expressions into equivalent forms. They determine appropriate techniques for solving each type of equation, inequality, or system of equations, apply the techniques correctly to solve, justify the steps in the solutions, and draw conclusions from the solutions. They know and apply common formulas.

A1.1 Construction, Interpretation, and Manipulation of Expressions (linear, quadratic, polynomial, rational, power, exponential, logarithmic, and trigonometric)

A1.1.1 Give a verbal description of an expression that is presented in symbolic form, write an algebraic expression from a verbal description, and evaluate expressions given values of the variables.

A1.1.2 Know the definitions and properties of exponents and roots, transition fluently between them, and apply them in algebraic expressions.

A1.1.3 Factor algebraic expressions using, for example, greatest common factor, grouping, and the special product identities (e.g., differences of squares and cubes).

A1.1.4 Add, subtract, multiply, and simplify polynomials and rational expressions (e.g., multiply $(x - 1)(1 - x^2 + 3)$; simplify $\frac{9x - x^3}{x + 3}$)

A1.1.5 Divide a polynomial by a monomial.

A1.1.6 Transform exponential and logarithmic expressions into equivalent forms using the properties of exponents and logarithms including the inverse relationship between exponents and logarithms.

A1.2 Solutions of Equations and Inequalities (linear, quadratic, polynomial, rational, power, exponential, logarithmic, and trigonometric)

A1.2.1 Write equations and inequalities with one or two variables to represent mathematical or applied situations, and solve.

A1.2.2 Associate a given equation with a function whose zeros are the solutions of the equation.

A1.2.3 Solve (and justify steps in the solutions) linear and quadratic equations and inequalities, including systems of up to three linear equations with three unknowns; apply the quadratic formula appropriately.

A1.2.4 Solve absolute value equations and inequalities, (e.g. solve $|x - 3| \leq 6$), and justify steps in the solution.

A1.2.5 Solve polynomial equations and equations involving rational expressions (e.g. solve $-2x(x^2 + 4x + 3) = 0$; solve $x - \frac{1}{x + 6} = 3$), and justify steps in the solution.

A1.2.6 Solve power equations (e.g., $(x + 1)^3 = 8$) and equations including radical expressions (e.g., $\sqrt{3x - 7} = 7$), justify steps in the solution, and explain how extraneous solutions may arise.

A1.2.7 Solve exponential and logarithmic equations (e.g., $3(2^x) = 24$), $2\ ln(x + 1) = 4$), and justify steps in the solution.

A1.2.8 Solve an equation involving several variables (with numerical or letter coefficients) for a designated variable, and justify steps in the solution.

A1.2.9 Know common formulas (e.g., slope, distance between two points, quadratic formula, compound interest, distance = velocity • time), and apply appropriately in contextual situations.

A1.2.10 Use special values of the inverse trigonometric functions to solve trigonometric equations over specific intervals (e.g., $2sin\ x - 1 = 0$ for $0 \leq x \leq 2\pi$).

STANDARD A2: FUNCTION

Students understand functions, their representations, and their attributes. They perform transformations, combine and compose functions, and find inverses. Students classify functions and know the characteristics of each family. They work with functions with real coefficients fluently.

A2.1 Definitions, Representations, and Attributes of Functions

A2.1.1 Recognize whether a relationship (given in contextual, symbolic, tabular, or graphical form) is a function; and identify its domain and range.

A2.1.2 Read, interpret, and use function notation, and evaluate a function at a value in its domain.

A2.1.3 Represent functions in symbols, graphs, tables, diagrams, or words, and translate among representations.

A2.1.4 Recognize that functions may be defined by different expressions over different intervals of their domains; such functions are piecewise-defined (e.g., absolute value and greatest integer functions).

A2.1.5 Recognize that functions may be defined recursively, and compute values of and graph simple recursively defined functions (e.g., $f(0) = 5$, and $f(n) = f(n\text{-}1) + 2$).

A2.1.6 Identify the zeros of a function and the intervals where the values of a function are positive or negative, and describe the behavior of a function, as x approaches postive or negative infinity, given the symbolic and graphical representations.

A2.1.7 Identify and interpret the key features of a function from its graph or its formula(e), (e.g. slope, intercept(s), asymptote(s), maximum and minimum value(s), symmetry, average rate of change over an interval, and periodicity).

A2.2 Operations and Transformations

A2.2.1 Combine functions by addition, subtraction, multiplication, and division.

A2.2.2 Apply given transformations (e.g., vertical or horizontal shifts, stretching or shrinking, or reflections about the x- and y-axes) to basic functions, and represent symbolically.

A2.2.3 Recognize whether a function (given in tabular or graphical form) has an inverse and recognize simple inverse pairs (e.g., $f(x) = x^3$ and $g(x) = x^{1/3}$).

A2.3 Families of Functions (linear, quadratic, polynomial, rational, power, exponential, logarithmic, and trigonometric)

A2.3.1 Identify a function as a member of a family of functions based on its symbolic, or graphical representation; recognize that different families of functions have different asymptotic behavior at infinity, and describe these behaviors.

A2.3.2 Describe the tabular pattern associated with functions having constant rate of change (linear); or variable rates of change.

A2.3.3 Write the general symbolic forms that characterize each family of functions. (e.g., $f(x) = A_0a^x$; $f(x) = A\sin Bx$)

A2.4 Lines and Linear Functions

A2.4.1 Write the symbolic forms of linear functions (standard [i.e., $Ax + By = C$, where $B \neq 0$], point-slope, and slope-intercept) given appropriate information, and convert between forms.

A2.4.2 Graph lines (including those of the form $x = h$ and $y = k$) given appropriate information.

A2.4.3 Relate the coefficients in a linear function to the slope and x- and y-intercepts of its graph.

A2.4.4 Find an equation of the line parallel or perpendicular to given line, through a given point; understand and use the facts that non-vertical parallel lines have equal slopes, and that non-vertical perpendicular lines have slopes that multiply to give -1.

A2.5 Exponential and Logarithmic Functions

A2.5.1 Write the symbolic form and sketch the graph of an exponential function given appropriate information. (e.g., given an initial value of 4 and a rate of growth of 1.5, write $f(x) = 4\ (1.5)^x$).

A2.5.2 Interpret the symbolic forms and recognize the graphs of exponential and logarithmic functions (e.g., $f(x) = 10^x$, $f(x) = \log x$, $f(x) = e^x$, $f(x) = ln\ x$); recognize the logarithmic function as the inverse of the exponential function.

A2.5.3 Apply properties of exponential and logarithmic functions (e.g., $a^{x+y} = a^x a^y$; $\log(ab) = \log a + \log b$).

A2.5.4 Understand and use the fact that the base of an exponential function determines whether the function increases or decreases and understand how the base affects the rate of growth or decay.

A2.5.5 Relate exponential and logarithmic functions to real phenomena, including half-life and doubling time.

A2.6 Quadratic Functions

A2.6.1 Write the symbolic form and sketch the graph of a quadratic function given appropriate information (e.g., vertex, intercepts, etc.).

A2.6.2 Identify the elements of a parabola (vertex, axis of symmetry, direction of opening) given its symbolic form or its graph, and relate these elements to the coefficient(s) of the symbolic form of the function.

A2.6.3 Convert quadratic functions from standard to vertex form by completing the square.

A2.6.4 Relate the number of real solutions of a quadratic equation to the graph of the associated quadratic function.

A2.6.5 Express quadratic functions in vertex form to identify their maxima or minima, and in factored form to identify their zeros.

A2.7 Power Functions (including roots, cubics, quartics, etc.)

A2.7.1 Write the symbolic form and sketch the graph of power functions.

A2.7.2 Express direct and inverse relationships as functions (e.g., $y = kx^n$ and $y = kx^{-n}$, $n > 0$) and recognize their characteristics (e.g., in $y = x^3$, note that doubling x results in multiplying y by a factor of 8).

A2.7.3 Analyze the graphs of power functions, noting reflectional or rotational symmetry.

A2.8 Polynomial Functions

A2.8.1 Write the symbolic form and sketch the graph of simple polynomial functions.

A2.8.2 Understand the effects of degree, leading coefficient, and number of real zeros on the graphs of polynomial functions of degree greater than 2.

A2.8.3 Determine the maximum possible number of zeros of a polynomial function, and understand the relationship between the x-intercepts of the graph and the factored form of the function.

A2.9 Rational Functions

A2.9.1 Write the symbolic form and sketch the graph of simple rational functions.

A2.9.2 Analyze graphs of simple rational functions (e.g., $f(x) = \frac{2x + 1}{x - 1}$; $g(x) = \frac{x}{x^2 - 4}$) and understand the relationship between the zeros of the numerator and denominator and the function's intercepts, asymptotes, and domain.

A2.10 Trigonometric Functions

A2.10.1 Use the unit circle to define sine and cosine; approximate values of sine and cosine (e.g., *sin 3*, or *cos 0.5*); use sine and cosine to define the remaining trigonometric functions; explain why the trigonometric functions are periodic.

A2.10.2 Use the relationship between degree and radian measures to solve problems.

A2.10.3 Use the unit circle to determine the exact values of sine and cosine, for integer multiples of $\pi/6$ and $\pi/4$.

A2.10.4 Graph the sine and cosine functions; analyze graphs by noting domain, range, period, amplitude, and location of maxima and minima.

A2.10.5 Graph transformations of basic trigonometric functions (involving changes in period, amplitude, and midline) and understand the relationship between constants in the formula and the transformed graph.

STANDARD A3: MATHEMATICAL MODELING

Students construct or select a function to model a real-world situation in order to solve applied problems. They draw on their knowledge of families of functions to do so.

A3.1 Models of Real-world Situations Using Families of Functions.

Example: An initial population of 300 people grows at 2% per year. What will the population be in 10 years?

A3.1.1 Identify the family of function best suited for modeling a given real-world situation (e.g., quadratic functions for motion of an object under the force of gravity; exponential functions for compound interest; trigonometric functions for periodic phenomena. *In the example above, recognize that the appropriate general function is exponential* ($P = P_0 a^t$)

A3.1.2 Adapt the general symbolic form of a function to one that fits the specifications of a given situation by using the information to replace arbitrary constants with numbers. *In the example above, substitute the given values* $P_0 = 300$ and $a = 1.02$ to obtain $P = 300(1.02)^t$.

A3.1.3 Using the adapted general symbolic form, draw reasonable conclusions about the situation being modeled. *In the example above, the exact solution is 365.698, but for this problem an appropriate approximation is 365.*

RECOMMENDED:

***A1.1.7** Transform trigonometric expressions into equivalent forms using basic identities such as $\sin^2\theta + \cos^2\theta = 1$, $\tan\theta = \frac{\sin\theta}{\cos\theta}$ and $\tan^2\theta + 1 = \sec^2\theta$

***A2.2.4** If a function has an inverse, find the expression(s) for the inverse.

***A2.2.5** Write an expression for the composition of one function with another; recognize component functions when a function is a composition of other functions.

***A2.2.6** Know and interpret the function notation for inverses and verify that two functions are inverses using composition.

***A3.1.4** Use methods of linear programming to represent and solve simple real-life problems.

STRAND 3: GEOMETRY AND TRIGONOMETRY (G)

In Grades K–5, students study figures such as triangles, rectangles, circles, rectangular solids, cylinders, and spheres. They examine similarities and differences between geometric shapes. They learn to quantify geometric figures by measuring and calculating lengths, angles, areas and volumes. In Grades 6-8, students broaden their understanding of area and volume and develop the basic concepts of congruence, similarity, symmetry and the Pythagorean Theorem. They apply these ideas to solve geometric problems, including ones related to the real world.

In Grades 9–12, students see geometry developed as a coherent, structured subject. They use the geometric skills and ideas introduced earlier, such as congruence and similarity, to solve a wide variety of problems. There is an emphasis on the importance of clear language (e.g. for postulates, definitions and theorems) and on learning to construct geometric proofs. In this process, students build geometric intuition and facility at deductive reasoning. They use elements of logic and reasoning as described in the Quantitative Literacy and Logic strand, including both direct and indirect proof presented in narrative form. They begin to use new techniques, including transformations and trigonometry. They apply these ideas to solve complex problems about two- and three-dimensional figures, again including ones related to the real world. Their spatial visualization skills will be developed through the study of the relationships between two- and three-dimensional shapes.

STANDARD G1: FIGURES AND THEIR PROPERTIES

Students represent basic geometric figures, polygons, and conic sections and apply their definitions and properties in solving problems and justifying arguments, including constructions and representations in the coordinate plane. Students represent three-dimensional figures, understand the concepts of volume and surface area, and use them to solve problems. They know and apply properties of common three-dimensional figures.

G1.1 Lines and Angles; Basic Euclidean and Coordinate Geometry

G1.1.1 Solve multi-step problems and construct proofs involving vertical angles, linear pairs of angles supplementary angles, complementary angles, and right angles.

G1.1.2 Solve multi-step problems and construct proofs involving corresponding angles, alternate interior angles, alternate exterior angles, and same-side (consecutive) interior angles.

G1.1.3 Perform and justify constructions, including midpoint of a line segment and bisector of an angle, using straightedge and compass.

G1.1.4 Given a line and a point, construct a line through the point that is parallel to the original line using straightedge and compass; given a line and a point, construct a line through the point that is perpendicular to the original line; justify the steps of the constructions.

G1.1.5 Given a line segment in terms of its endpoints in the coordinate plane, determine its length and midpoint.

G1.1.6 Recognize Euclidean Geometry as an axiom system; know the key axioms and understand the meaning of and distinguish between undefined terms (e.g., point, line, plane), axioms, definitions, and theorems.

G1.2 Triangles and Their Properties

G1.2.1 Prove that the angle sum of a triangle is 180° and that an exterior angle of a triangle is the sum of the two remote interior angles.

G1.2.2 Construct and justify arguments and solve multi-step problems involving angle measure, side length, perimeter, and area of all types of triangles.

G1.2.3 Know a proof of the Pythagorean Theorem and use the Pythagorean Theorem and its converse to solve multi-step problems.

G1.2.4 Prove and use the relationships among the side lengths and the angles of 30°- 60°- 90° triangles and 45°- 45°- 90° triangles.

G1.2.5 Solve multi-step problems and construct proofs about the properties of medians, altitudes, and perpendicular bisectors to the sides of a triangle, and the angle bisectors of a triangle; using a straightedge and compass, construct these lines.

G1.3 Triangles and Trigonometry

G1.3.1 Define the sine, cosine, and tangent of acute angles in a right triangle as ratios of sides; solve problems about angles, side lengths, or areas using trigonometric ratios in right triangles.

G1.3.2 Know and use the Law of Sines and the Law of Cosines and use them to solve problems; find the area of a triangle with sides a and b and included angle θ using the formula Area = (1/2) $a\ b \sin\theta$.

G1.3.3 Determine the exact values of sine, cosine, and tangent for 0°, 30°, 45°, 60°, and their integer multiples, and apply in various contexts.

G1.4 Quadrilaterals and Their Properties

G1.4.1 Solve multi-step problems and construct proofs involving angle measure, side length, diagonal length, perimeter, and area of squares, rectangles, parallelograms, kites, and trapezoids.

G1.4.2 Solve multi-step problems and construct proofs involving quadrilaterals (e.g., prove that the diagonals of a rhombus are perpendicular) using Euclidean methods or coordinate geometry.

G1.4.3 Describe and justify hierarchical relationships among quadrilaterals, (e.g. every rectangle is a parallelogram).

G1.4.4 Prove theorems about the interior and exterior angle sums of a quadrilateral.

G1.5 Other Polygons and Their Properties

G1.5.1 Know and use subdivision or circumscription methods to find areas of polygons (e.g., regular octagon, non-regular pentagon).

G1.5.2 Know, justify, and use formulas for the perimeter and area of a regular n-gon and formulas to find interior and exterior angles of a regular n-gon and their sums.

G1.6 Circles and Their Properties

G1.6.1 Solve multi-step problems involving circumference and area of circles.

G1.6.2 Solve problems and justify arguments about chords (e.g., if a line through the center of a circle is perpendicular to a chord, it bisects the chord) and lines tangent to circles (e.g., a line tangent to a circle is perpendicular to the radius drawn to the point of tangency).

G1.6.3 Solve problems and justify arguments about central angles, inscribed angles and triangles in circles.

G1.6.4 Know and use properties of arcs and sectors, and find lengths of arcs and areas of sectors.

G1.7 Conic Sections and Their Properties

G.1.7.1 Find an equation of a circle given its center and radius; given the equation of a circle, find its center and radius.

G1.7.2 Identify and distinguish among geometric representations of parabolas, circles, ellipses, and hyperbolas; describe their symmetries, and explain how they are related to cones.

G1.7.3 Graph ellipses and hyperbolas with axes parallel to the x- and y-axes, given equations.

G1.8 Three- Dimensional Figures

G1.8.1 Solve multi-step problems involving surface area and volume of pyramids, prisms, cones, cylinders, hemispheres, and spheres.

G1.8.2 Identify symmetries of pyramids, prisms, cones, cylinders, hemispheres, and spheres.

STANDARD G2: RELATIONSHIPS BETWEEN FIGURES

Students use and justify relationships between lines, angles, area and volume formulas, and 2- and 3-dimensional representations. They solve problems and provide proofs about congruence and similarity.

G2.1 Relationships Between Area and Volume Formulas

G2.1.1 Know and demonstrate the relationships between the area formula of a triangle, the area formula of a parallelogram, and the area formula of a trapezoid.

G2.1.2 Know and demonstrate the relationships between the area formulas of various quadrilaterals (e.g., explain how to find the area of a trapezoid based on the areas of parallelograms and triangles).

G2.1.3 Know and use the relationship between the volumes of pyramids and prisms (of equal base and height) and cones and cylinders (of equal base and height).

G2.2 Relationships Between Two-dimensional and Three-dimensional Representations

G2.2.1 Identify or sketch a possible 3-dimensional figure, given 2-dimensional views (e.g., nets, multiple views); create a 2-dimensional representation of a 3-dimensional figure.

G2.2.2 Identify or sketch cross-sections of 3-dimensional figures; identify or sketch solids formed by revolving 2-dimensional figures around lines.

G2.3 Congruence and Similarity

G2.3.1 Prove that triangles are congruent using the SSS, SAS, ASA, and AAS criteria, and for right triangles, the hypotenuse-leg criterion.

G2.3.2 Use theorems about congruent triangles to prove additional theorems and solve problems, with and without use of coordinates.

G2.3.3 Prove that triangles are similar by using SSS, SAS, and AA conditions for similarity.

G2.3.4 Use theorems about similar triangles to solve problems with and without use of coordinates.

G2.3.5 Know and apply the theorem stating that the effect of a scale factor of k relating one two dimensional figure to another or one three dimensional figure to another, on the length, area, and volume of the figures is to multiply each by k, k^2, and k^3, respectively.

STANDARD G3: TRANSFORMATIONS OF FIGURES IN THE PLANE

Students will solve problems about distance-preserving transformations and shape-preserving transformations. The transformations will be described synthetically and, in simple cases, by analytic expressions in coordinates.

G3.1 Distance-preserving Transformations: Isometries

G3.1.1 Define reflection, rotation, translation, and glide reflection and find the image of a figure under a given isometry.

G3.1.2 Given two figures that are images of each other under an isometry, find the isometry and describe it completely.

G3.1.3 Find the image of a figure under the composition of two or more isometries, and determine whether the resulting figure is a reflection, rotation, translation, or glide reflection image of the original figure.

G3.2 Shape-preserving Transformations: Dilations and Isometries

G3.2.1 Know the definition of dilation, and find the image of a figure under a given dilation.

G3.2.2 Given two figures that are images of each other under some dilation, identify the center and magnitude of the dilation.

RECOMMENDED:

***G1.4.5** Understand the definition of a cyclic quadrilateral, and know and use the basic properties of cyclic quadrilaterals.

***G1.7.4** Know and use the relationship between the vertices and foci in an ellipse, the vertices and foci in a hyperbola, and the directrix and focus in a parabola; interpret these relationships in applied contexts.

***G3.2.3** Find the image of a figure under the composition of a dilation and an isometry.

STRAND 4: STATISTICS AND PROBABILITY (S)

In Kindergarten through Grade 8, students develop the ability to read, analyze, and construct a repertoire of statistical graphs. Students also examine the fundamentals of experimental and theoretical probability in informal ways. The Basic Counting Principle and tree diagrams serve as tools to solve simple counting problems in these grades.

During high school, students build on that foundation. They develop the data interpretation and decision-making skills that will serve them in their further study of mathematics as well as in their coursework in the physical, biological, and social sciences. Students learn important skills related to the collection, display, and interpretation of both univariate and bivariate data. They understand basic sampling methods and apply principles of effective data analysis and data presentation. These skills are also highly valuable outside of school, both in the workplace and in day-to-day life.

In probability, students utilize probability models to calculate probabilities and make decisions. The normal distribution and its properties are studied. Students then use their understanding of probability to make decisions, solve problems, and determine whether or not statements about probabilities of events are reasonable. Students use technology when appropriate, including spreadsheets. This strong background in statistics and probability will enable students to be savvy decision-makers and smart information-consumers and producers who have a full range of tools in order to make wise choices.

STANDARD S1: UNIVARIATE DATA – EXAMINING DISTRIBUTIONS

Students plot and analyze univariate data by considering the shape of distributions and analyzing outliers; they find and interpret commonly-used measures of center and variation; and they explain and use properties of the normal distribution.

S1.1 Producing and Interpreting Plots

S1.1.1 Construct and interpret dot plots, histograms, relative frequency histograms, bar graphs, basic control charts, and box plots with appropriate labels and scales; determine which kinds of plots are appropriate for different types of data; compare data sets and interpret differences based on graphs and summary statistics.

S1.1.2 Given a distribution of a variable in a data set, describe its shape, including symmetry or skewness, and state how the shape is related to measures of center (mean and median) and measures of variation (range and standard deviation) with particular attention to the effects of outliers on these measures.

S1.2 Measures of Center and Variation

S1.2.1 Calculate and interpret measures of center including: mean, median, and mode; explain uses, advantages and disadvantages of each measure given a particular set of data and its context.

S1.2.2 Estimate the position of the mean, median, and mode in both symmetrical and skewed distributions, and from a frequency distribution or histogram.

S1.2.3 Compute and interpret measures of variation, including percentiles, quartiles, interquartile range, variance, and standard deviation.

S1.3 The Normal Distribution

S1.3.1 Explain the concept of distribution and the relationship between summary statistics for a data set and parameters of a distribution.

S1.3.2 Describe characteristics of the normal distribution, including its shape and the relationships among its mean, median, and mode.

S1.3.3 Know and use the fact that about 68%, 95%, and 99.7% of the data lie within one, two, and three standard deviations of the mean, respectively in a normal distribution.

S1.3.4 Calculuate z-scores, use z-scores to recognize outliers, and use z-scores to make informed decisions.

STANDARD S2: BIVARIATE DATA – EXAMINING RELATIONSHIPS

Students plot and interpret bivariate data by constructing scatterplots, recognizing linear and nonlinear patterns, and interpreting correlation coefficients; they fit and interpret regression models, using technology as appropriate.

S2.1 Scatterplots and Correlation

S2.1.1 Construct a scatterplot for a bivariate data set with appropriate labels and scales.

S2.1.2 Given a scatterplot, identify patterns, clusters, and outliers; recognize no correlation, weak correlation, and strong correlation.

S2.1.3 Estimate and interpret Pearson's correlation coefficient for a scatterplot of a bivariate data set; recognize that correlation measures the strength of linear association.

S2.1.4 Differentiate between correlation and causation; know that a strong correlation does not imply a cause-and-effect relationship; recognize the role of lurking variables in correlation.

S2.2 Linear Regression

S2.2.1 For bivariate data which appear to form a linear pattern, find the least squares regression line by estimating visually and by calculating the equation of the regression line; interpret the slope of the equation for a regression line.

S2.2.2 Use the equation of the least squares regression line to make appropriate predictions.

STANDARD S3: SAMPLES, SURVEYS, AND EXPERIMENTS

Students understand and apply sampling and various sampling methods, examine surveys and experiments, identify bias in methods of conducting surveys, and learn strategies to minimize bias. They understand basic principles of good experimental design.

S3.1 Data Collection and Analysis

S3.1.1 Know the meanings of a sample from a population and a census of a population, and distinguish between sample statistics and population parameters.

S3.1.2 Identify possible sources of bias in data collection and sampling methods and simple experiments; describe how such bias can be reduced and controlled by random sampling; explain the impact of such bias on conclusions made from analysis of the data; and know the effect of replication on the precision of estimates.

S3.1.3 Distinguish between an observational study and an experimental study, and identify, in context, the conclusions that can be drawn from each.

STANDARD S4: PROBABILITY MODELS AND PROBABILITY CALCULATION

Students understand probability and find probabilities in various situations, including those involving compound events, using diagrams, tables, geometric models and counting strategies; they apply the concepts of probability to make decisions.

S4.1 Probability

S4.1.1 Understand and construct sample spaces in simple situations (e.g., tossing two coins, rolling two number cubes and summing the results).

S4.1.2 Define mutually exclusive events, independent events, dependent events, compound events, complementary events and conditional probabilities; and use the definitions to compute probabilities.

S4.2 Application and Representation

S4.2.1 Compute probabilities of events using tree diagrams, formulas for combinations and permutations, Venn diagrams, or other counting techniques.

S4.2.2 Apply probability concepts to practical situations, in such settings as finance, health, ecology, or epidemiology, to make informed decisions.

RECOMMENDED:

***S3.1.4** Design simple experiments or investigations to collect data to answer questions of interest; interpret and present results.

***S3.1.5** Understand methods of sampling, including random sampling, stratified sampling, and convenience samples, and be able to determine, in context, the advantages and disadvantages of each.

***S3.1.6** Explain the importance of randomization, double-blind protocols, replication, and the placebo effect in designing experiments and interpreting the results of studies.

***S3.2.1** Explain the basic ideas of statistical process control, including recording data from a process over time.

***S3.2.2** Read and interpret basic control charts; detect patterns and departures from patterns.

***S4.1.3** Design and carry out an appropriate simulation using random digits to estimate answers to questions about probability; estimate probabilities using results of a simulation; compare results of simulations to theoretical probabilities.

ADDENDUM

Additional Mathematics Recommended for All Students

In addition to the core and recommended expectations, we provide here a set of fundamental topics in the areas of precalculus and statistics and probability. Study in these areas is recommended as preparation for post-secondary education opportunities, as well as for the workplace.

Precalculus

Precalculus is the preparation for calculus. Calculus is the most powerful, useful and versatile branch of mathematics. While the core ideas of calculus (derivatives and integrals) are not hard to understand, calculus is a demanding subject because it requires a broad and thorough background of algebra and functions. This material is essential for college-bound students. It is a prerequisite for many college programs in science, engineering, medicine, and business.

Students study precalculus in order to deeply understand, make connections among, and apply the topics introduced in algebra. Those algebraic topics are now treated at a higher conceptual level. The theory and applications of trigonometry and functions are developed in depth. New mathematical tools, such as vectors, matrices and polar coordinates, are introduced with an eye toward modeling and solving real world problems.

- ***Functions***

  Students use the definition of function (including domain and range). They combine functions using algebraic operations and composition. They write a given function as a composition of simpler functions. The notion of one-to-one function is introduced, leading to the definition of an inverse function. Students find the symbolic expression for the inverse of a given function and show that two given functions are inverses.

- ***Exponential and logarithmic functions***

  Students graph logarithmic functions as inverses of exponential functions, and solve equations involving exponential and logarithmic functions. They determine the asymptotic behavior of exponential and logarithmic functions with different bases. Students apply these functions in real world situations, such as exponential growth and decay, and compound interest.

- ***Trigonometric functions and analytical trigonometry***

  Students use the unit circle to define sine, cosine, and the other trigonometric functions. They apply transformations involving changes in amplitude, midline, period, and phase, to trigonometric functions and represent the results graphically and symbolically. The inverse trigonometric functions and their graphs are introduced. They establish and learn sum and difference formulas and other basic trigonometric identities. They use these to simplify trigonometric expressions, to solve trigonometric equations, prove trigonometric identities, and solve applied problems.

- ***Polynomial and rational functions***

  Students learn the Fundamental Theorem of Algebra, the Remainder Theorem, and the Factor Theorem. They solve polynomial equations and inequalities by factoring and dividing polynomials. They identify the large-scale behavior of the graph of a polynomial from its leading term. Students solve rational equations and inequalities. They determine the asymptotes of the graph of a rational function.

ADDENDUM (CONT.)

- ***Difference quotients and limits***

  Students learn the definition and geometric interpretation of difference quotients. Using the definition, they represent and simplify difference quotients and interpret difference quotients as rates of change and slopes of secant lines. They acquire an informal meaning for the limit of a function and relate that meaning to the graph.

- ***Vectors and matrices***

  Students sketch and perform operations (multiplication by scalars, addition, and subtraction) of vectors in the plane and use vectors in applications. They learn the algebraic and geometric definitions of dot product of vectors, and use them in applications. Students represent rotations of the plane as matrices and apply these in the context of analytic geometry. They multiply matrices and multiply vectors by matrices, compute determinants, and they solve systems of two and three linear equations by matrix methods. Students compute inverses of three-by-three matrices, when they exist.

- ***Sequences and series***

  Students find the nth term in arithmetic sequences, geometric sequences, and recursively defined sequences. They use sigma notation and compute sums of finite arithmetic sequences. Students compute sums of finite and infinite geometric series and apply the convergence criterion for geometric series.

- ***Polar coordinates, parameterizations, and conic sections***

  Students use polar coordinates and graph equations in polar form. They write complex numbers in polar form and use DeMoivre's Theorem. Students parameterize segments and curves, and recognize implicitly defined curves. They identify parabolas, ellipses and hyperbolas from their equations, put the equations in standard form, sketch and analyze the graphs and characteristics (e.g. finding foci).

- ***Mathematical reasoning***

  Students prove theorems and use mathematical induction.

Statistics and Probability

Students encounter variability in their lives, in their science and social studies coursework, and in the news media. The study of statistics and probability gives students methods for summarizing data, introduces students to mathematical models for random phenomena, and provides the tools for decision making under uncertainty.

Three important considerations should be kept in mind when reading the topics below. First, as much as possible, the concepts and techniques should be introduced and used in the context of specific studies. Sometimes these are called "real-world" applications, although it is often useful to clean up data before presenting it to students, so that the essential concepts are not obscured by the complexities of the data. Second, wherever possible, technology (calculators or statistical software) should be used for computations and graphing. Third, simulation can give students deeper understanding of many probability and inferential concepts, and should be used often.

The topics are built around four themes: data exploration and study design; probability models and their application; statistical inference; and model assessment. Some of the topics have been introduced in the required curriculum, and are extended and treated in more depth in the extended curriculum. More details on the specific topics are given next.

- ***Exploring univariate and bivariate data***

  Students learn and apply techniques for exploring univariate and bivariate data using both graphical and numerical summaries. Fundamental is fostering students' understanding of variability in data, and learning how to make comparisons between data sets in the presence of variability.

- ***Sampling and study design***

  Students learn methods of designing surveys and controlled experiments, design surveys and experiments, and use their knowledge of design to critically assess conclusions. The importance of randomization in minimizing bias, and of methods such as blocking to reduce variability, are stressed.

- ***Probability models***

  Students are introduced to commonly-used discrete probability models such as the binomial and hypergeometric models and use these to model real-world phenomena. Conditional probabilities including Bayes' Theorem are used to solve problems from health, public policy, and other areas, and students gain facility with the normal distribution.

- ***Sampling distributions***

  Students learn results for sums of random variables, including an informal treatment of the central limit theorem, and apply these results to sampling distributions of common estimators. Statistical process control and the creation and interpretation of control charts are included.

- ***Point and interval estimation***

  Students study basic properties of point estimators, and then apply their knowledge of sampling distributions to construct confidence interval estimators for means and proportions in one- and two- sample problems for both means and proportions. Correct interpretation of confidence intervals is stressed.

- ***Significance testing***

  Students learn the terminology and logic of significance testing, including power, learn to perform significance tests for means and proportions in one- and two-sample problems for means and proportions, and are introduced to chi-squared testing. A main focus is on correct interpretation of results.

- ***Inference for regression***

  The statistical model for simple linear regression is introduced, and students learn to construct confidence intervals and perform significance tests for the slope of a regression line.

- ***Assessing assumptions of statistical models***

  Throughout the sections on probability and statistical inference, the role of the underlying mathematical model and its assumptions is kept in the forefront. Students learn to assess the validity of the model and to gauge the effect of departures from model assumptions.